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TRENDS OF MATHEMATICAL LOGICAL THINKING ABILITY THROUGH THE CRA (CONCRETE-REPRESENTATIONAL-ABSTRACT) APPROACH WITH PORTFOLIO ASSESSMENT

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Abstract

The use of an approach in learning can provide a change in pre-service teachers mathematical logical thinking abilities, this study aims to look at the trends of mathematical logical thinking ability with treatments using Concrete-Representational-Abstract (CRA) approach with portfolio assessment. This research is a quasi-experimental with time series design. That has been acquired by new pre-service teachers. The subject of this study consists of 37 first year undergraduate students in one of the universities in Banten province, Indonesia. Pretest and posttest data were analyzed using paired sample t-test. The instruments consisted of 4 packages of mathematical logical thinking ability for pretest and posttest with each instrument consists of 8 questions on 4 indicators. The conclusion shows that there are significant differences and improvements in mathematical logical thinking ability from time to time after treatments in learning with CRA. The development of pre-service teachers mathematical logical thinking ability after CRA has trends to increase above the trendline with the equation y = 0.5085x - 1.4316.

Keyword: *Concrete-Representational-Abstract (CRA), logical thinking, portfolio.*

Activities requiring such critical skills as problem-solving, creative, critical and reflective thinking would also improve teacher candidates' logical thinking ability (Tuna, Biber, & Incikapi, 2013). In addition, one cognitive skill that increases academic success is the ability to think logically (Yaman, 2005). The ability to think logically about individual skills to solve problems by using their ability to achieve principles or rules by making generalizations or abstractions. The characteristics of logical thinking according to Ni'matus (Andriawan & Budiarto, 2014) include: (a) thinking chaos, which is compiled from the beginning of planning to conclusions that support the steps that are recommended, (b) the ability to argue, that is logical and in accordance with the facts or information available can provide an argument related to the problem planning steps and discussion of the issues discussed, and (c) discussion, namely from where can draw conclusions based on the steps that have been discussed. There are several important things about mathematical logical thinking ability developed in mathematics learning, namely that the need to develop reasoning and logical thinking ability in mathematics learning because it can improve abilities in mathematics which used to be just to remember understanding skills (Mukhayat, 2004; Sumarmo, Hidayat, Zukarnaen, Hamidah, & Sariningsih, 2012). Logical thinking: processes can make people "smarter" (Edublox, 2006). The ability to think logically using realistic and small group mathematical approaches is significantly better than those with learning using ordinary learning, learning using electric circuits in the material mathematical logical can improve pre-service teachers logical thinking ability at Raden Intan Lampung IAIN (Netriwati, 2015; Saragih, 2017).

Development of thinking ability starts from the development of cognitive theory, this very well-known theory proposed by Jean Piaget's (Yoon Fah, 2009) has conceptualized four different

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stages in a person's cognitive development namely sensorimotor (0-2 years), preoperational (2-7 years), concrete operational (7-11 years) and formal operational (11-16 years). The main difference between the stages of cognitive development is the way of thinking. If you look at it in terms of age at the formal operational stage, you can think logically about abstract propositions and test hypotheses systematically. At the same time, they are doing hypothetically, guessing results and solving problems. Piaget (Sezen & Bülbül, 2011) defines logical thinking abilities observed in concrete stages and abstract stages of operations. At the stage of concrete operations, students can use the ability to think logically in solving the problems that are concrete while at the stage of abstract operations, students reach the adult level in terms of logical thinking. The most important way of thinking from concrete thinking to the formal operational stage is logical thinking ability, logical thinking ability are considered high cognitive skills, and they can function in Piaget's cognitive development stage that cannot emerge before the concrete operational stage (Atherson, n.d.; Minderovic, n.d.). Logical words we often hear in everyday life, logical thinking is closely associated with logic or sometimes someone uses when hearing the opinions of others is not in accordance with the decision making (reasonable) of a problem that is said to be illogical. This means that the logical word contains certain rules that must be fulfilled. Logical thinking is a skill that is determined in the stages of Piaget's cognitive development process, with the ability to think logically, students solve problems by carrying out various practices and achieving principles or rules by doing some abstractions and generalizations (Yaman, 2005). Khin Mar Ni said that logical thinking is thinking in terms of cause and effect, which means thinking sequentially (Yin, Wuttye, & Yee, n.d.).

Whereas Sponias (Sumarmo et al., 2012) defines thinking as a process based on ideas that try to understand reality and find solutions for various problems (thinking is a process based on thoughts of reality and final understandings to various problems and say logic is a collection of thoughts that judge reality. Logical words contain great or precise meanings based on thinking rules and general rules or standards that can be used to be able to think right (Mukhayat, 2004). Whereas in mathematics the logical word is closely related to the use of logic rules. The reasoning is a thought process that produces knowledge so that the knowledge generated by reasoning has the basis of truth, the thinking process must be done in a certain way (Suriasumantri, 2014). A new conclusion is considered valid if the conclusion is done in a certain way, how to draw conclusions is called logic. According to Sahakian (Suriasumantri, 2014) logic is defined as a study to think legitimately. Poedjawijatna said that people who think logically will obey according to the rules of logic. Plato said that thinking is speaking in the heart, or Gieles said that thinking is talking to himself inwardly, namely considering, pondering, analyzing, proving something, showing reasons, drawing conclusions, examining things, thinking about things it relates to each other (Mukhayat, 2004).

Thinking can be said to be a process to find a truth or true knowledge by involving the knowledge or experience possessed. Truth or true knowledge has a single meaning, which is diverse is the process of finding the truth or finding true knowledge (Saragih, 2017). Thus it can be stated that the truth or true knowledge will be obtained as long as the thought process is correct in the true sense according to the principles, laws, and rules. If doing thinking activities in accordance with what is mentioned above, then a scientific discipline arises about the right process of thinking, namely logic. In logic, learned the rules that must be held so that the thinking process is valid. To understand the logic, it must have a clear understanding of reasoning, because reasoning is a thought process that refers to laws or rules of logic. Thus it can be said that reasoning is a process of logical thinking. According to Demirel (Bakir & Oztekin-Bicer, 2015), logical thinking includes thinking effectively in using numbers, finding scientific solutions to a problem, realizing the differences between concepts, classifications, making generalizations and calculations, and giving hypotheses. Logical thinking is the ability to think of students to draw legitimate conclusions according to the rules of logic and can prove that conclusions are true (valid) in accordance with previous knowledge that is already known (Syaiful, 2011). Logical thinking is a thought process that uses reasoning consistently to produce conclusions (Sumarto, 2006). Problems or situations that involve logical thinking require structures, relationships between facts, arguments and understandable sets of reasoning, logical thinking ability are skills students have in expressing a truth based on facts (Sumarmo, 2002; Sumarmo et al., 2012). According to Charan (Sumarmo et al., 2012), there are five elements of logical thinking, namely: controlling variables, hypothesis reasoning, sequence reasoning, combinatorial reasoning, and correlation reasoning. There are five main characteristics of logical thinking, namely as follows: a) Proportional reasoning is the ability to determine and compare ratios, b) Controlling variables is the ability to plan, implement and interpret information, c) Probability reasoning is the ability to interpret the data obtained in the form of the magnitude of the possibility of an event, d) Correlational reasoning is the ability to determine whether two events or variables are interconnected or not, e) Combinatorial reasoning is the ability to determine the combination of an event (Tobin & Capie, 1981). In addition Yoon Fah stated that various researchers (Inhelder & Piaget, 1958; Rudner, Boston, Leydens, & Mehrens, 2012) have identified five different modes of operational formal reasoning namely proportional reasoning, controlling variables, probabilistic reasoning, correlational reasoning and combination reasoning which determines the success of students in science and mathematics at the secondary level.

Logical thinking is inseparable from the basis of reality because what is thought is a reality, namely the law of reality that is in harmony with the rules of thought. According to Kant (Tafsir, 2004) distinguishes between rational and logical, where rational is a rational thought, measured by natural law, while logical is a reasonable thought whose truth relies on arguments and is not measured by natural law. This implies that in a logical word a certain rule must be fulfilled so that it gets the correct conclusion. According to Albrecht (Syaiful, 2011) for someone to think logically, they must understand the logic of (a) the rational or reality, (b) arguments of shared thinking, and (c) conclusions or results achieved with the rational. In mathematics, the process of obtaining truth rationally or the process of drawing conclusions can be done by deductive and inductive thinking. Whereas proportional thinking, combinatoric thinking, controlling variables, and thinking probabilities develop at the stage of formal operations (Inhelder & Piaget, 1958). At this stage, pre-service teachers have been able to deduce a truth based on proportions, combinatorics, controlling variables and probabilities. This means that at the formal operating stage pre-service teachers must have the ability to think logically. At the stage of concrete development, pre-service teachers can only know mathematical symbols but have not been able to deal with abstract things. Combinatoric thinking is the ability to draw conclusions by considering all possible alternatives in a particular situation. Formal operations when solving problems will use all possible combinations or factors that are related to the problem. While thinking probability is the ability to draw conclusions relating to the data obtained in the form of the possibility of occurrence of an event. Probabilistic thinking will make a person distinguish things that happen and things that might occur based on the calculation of opportunities. Correlational thinking is the ability to analyze and draw conclusions about the strength of the reciprocal relationship between two variables and explain the principles in the relationship. Correlational thinking involves identifying and verifying relationships between variables. The ability to analyze the strength of the relationships between variables gives the right contribution to draw conclusions.

Based on the above description and the characteristics of the students it can be concluded that the ability to think logically in mathematics in this study is defined as an ability to use rules, traits or mathematical logic to get a correct conclusion. The ability to make generalizations and conclusions based on proportional thinking, probability thinking, correlational thinking, and combinatoric thinking. These four characteristics illustrate the ability of a person to think logically in solving mathematical problems related to the lecture material of the elementary school. So that pre-service teachers need to have logical thinking ability as prospective teachers through activities that require critical ability including problem-solving, creative thinking, critical and reflective. The more often involved in problem-solving activities, the better the logical thinking ability of pre-service teachers will be, so to achieve these goals a lecturer should providing learning in using a particular method or approach. What approach is suitable for forming abilities in thinking then appears as a question based on the explanation above which can involve all activities optimally, and make mathematics lessons in lectures meaningful and enjoyable. A learning approach plays an important role to improve pre-service teachers abilities, there are several approaches in mathematics learning including contextual approaches, constructivism approaches, RME (Realistic Mathematics Education) approaches, scientific approaches, Open-Ended Problem approaches, Concrete-Representational-Abstract (CRA) approaches, and etc. From the various approaches, the CRA approach was chosen because the CRA (Concrete Representational Abstract) approach was presented as a learning approach that was carried out in stages in accordance with the pre-service teachers ability. In addition, in order to improve pre-service teachers mathematical logical thinking ability there was a learning process that emphasized active pre-service teachers learning methods, by going through the stages in the CRA approach can equip pre-service teachers with logical thinking ability so that they are expected and apply it to various disciplines. In addition, pre-service teachers are expected to solve problems related to mathematics in life, because in a meaningful context mathematics must be studied by relating it to other subjects based on the experiences and interests of pre-service teachers. So need for thinking about mathematics learning that could improvement and development of pre-service teachers logical thinking ability. This CRA approach teaches students to learn based on three stages, namely: Concrete, Representational, Abstract. The learning process with CRA goes through three stages where students solve mathematical problems through concrete physical objects that coincide with learning activities through pictorial representation of concrete, and ends with solving mathematical problems with abstract notations such as numbers and symbols (Witzel, B., Ferguson & Mink, 2012; Witzel, Mercer, & Miller, 2003; Witzel, Riccomini, & Schneider, 2008). Another term that has been used to describe concrete teachings to semi-concrete, then to abstract. In Singapore, this approach is better known as the CPA (Concrete-Pictorical-Abstract) approach. The learning process with the CRA approach consists of stages that are believed to be the stages needed in the ability to think logically. The ability to think logically is the ability to be able to connect between concrete problems, in this case, the problems in everyday life, then represented into an abstract form that is into a mathematical form.

According to Bruner (Hudoyo, 1990) so that the process of learning a knowledge or an ability takes place optimally, in the sense that knowledge and abilities can be internalized in the cognitive structure of the person concerned, the learning process must go through three stages namely enactive (concrete), iconic (semi-concrete) and symbolic (abstract). Whereas Alimin (Hudoyo, 1990) states that there are four hierarchical learning steps that can be determined in mathematics learning, namely learning at concrete stages, semi-concrete stages, semi-abstract stages, and abstract stages. Witzel also explained that one of the lessons that use systematic learning stages such as learning is the CRA approach, Concrete, Representational, Abstract (Witzel et al., 2008). CRA's approach to the learning process starts from concrete things using appropriate concrete objects then continues with understanding representations with the help of appropriate images, and finally, the knowledge that has been known in advance from the two stages is used in the next stage, namely the abstract understanding stage.

According to Jane CRA is an approach or intervention in mathematics learning where the results of research showing through CRA can improve pre-service teachers ability and performance in learning mathematics. CRA consists of three stages with each stage to support each other's previous and subsequent stages, to support pre-service teachers learning and reduce pre-service teachers mistakes in understanding mathematical conceptual knowledge. According to Steedly, Dragoo, Arafeh, and Luke CRA are the most common examples in mathematics learning to combine visual representation. The three stages in CRA are 1) Concrete, the lecturer starts learning by modeling each mathematical concept with concrete material or objects such as cards colored beads (yellow, red), build cubes, beams and blocks, and other geometric shapes. At this stage, it is the most important stage to develop a conceptual understanding of mathematical concepts or ability. This learning stage occurs through the actions of pre-service teachers directly seen in manipulating objects in this case real objects. pre-service teachers directly hold concrete objects and use them, actually building a mental image and reality that is physically explored. Lecturers who will use concrete stages can start the learning process by modeling mathematical concepts with concrete objects that are around pre-service teachers, not with something abstract or outside.

Therefore, it is very basic if in the learning process the lecturer associates material with the life experience of pre-service teachers so they can understand mathematics as something that is experienced and fun, realize, and mathematics is close to the daily lives, then after the first stage, pre-service teachers switch to the second stage of learning activities, taken from concrete learning to the representation learning stage. 2) Representational, at this stage the lecturer changes the concrete model to the level of representation (semi-concrete) that may involve images, using circles, dots, and calculations, attaching something to help in counting. This stage converts manipulative objects (concrete) with images or depicts objects, this part which is from semi-concrete to semi-abstract (Carmichael, Ramadan, & Gaines-Montgomery, 2016; D. Montgomery, 2008; D. C. Montgomery, 2001). Pre-service teachers at the stage of representation can re-imagine or give an idea in his mind about the object or event that he experienced or what he knew at a concrete stage, even though the event had passed or the concrete object (real) was no longer in front of him. Knowledge is represented (manifested) in the form of visual imagery that may involve images, using circles, dots, lines, diagrams, and graphs, which describe concrete activities or concrete situations that are present in the previous concrete stages. The pictures made by pre-service teachers represent concrete objects manipulated when solving problems at a concrete stage. Pre-service teachers start drawing solutions to a problem as soon as they can show that they have mastered certain concepts or ability at a concrete stage.

Lecturers must provide many opportunities for pre-service teachers to practice and demonstrate concepts or mathematical ability. Drawing is not helped will do forever. It is merely an effective way to practice solving problems independently until they are able to develop ability at an abstract level. 3) Abstract, at this stage models mathematical concepts with symbols, mathematical concepts are modeled in forms of abstract symbols, namely symbols that are used based on the agreement of people in the field concerned, both symbols verbal symbols (eg letters, words, sentences), mathematical symbols (using numbers, notations and symbols), as well as other abstract symbols. Used operation symbols (+, -, *, :) to indicate addition, subtraction, multiplication and division operations. This stage is manipulating symbols or images of objects such as at the concrete stage and the representation stage. Some opportunities and demonstrations must be provided for pre-service teachers to achieve mastery of mathematical concepts (Carmichael et al., 2016; D. Montgomery, 2008; D. C. Montgomery, 2001).

Abstract understanding is often referred to as "doing math in your head". Understanding symbols and explaining with the language then working on mathematical questions in writing and students solving this problem in writing is also a common example of problem-solving in the abstract stage, besides being able to explain verbally how to solve a problem (Rahmawati & Nuraeni, 2015). The sequence in learning the CRA approach supports the conceptual students to form connections or meaningful relationships in understanding between concrete, representational, abstract. CRA works with "hands-on material" which represents mathematical problems (concrete), pictorial representations of mathematical problems (representations), and mathematical problems with numbers and symbols (abstract). CRA facilitating to reflect and analyze pre-service teachers work, with the stages of CRA allowing lecturers to observe classroom understanding as pre-service teachers demonstrate their understanding and ability possessed. The lectures explicitly bridge and facilitates the relationship between concrete, representations, and abstracts of mathematical problems (Arroyo, 2014; Powell & Seethaler, 2013).

In addition, CRA also targets visual, tactile, kinesthetic, or logical learning styles in structure, as well as, benefiting all grade levels and in populations (Witzel, B., Ferguson & Mink, 2012; Witzel et al., 2008). Starting with visual experience, tactile and kinesthetic experience to build understanding, broaden their understanding with representations of images of concrete objects and move to the level of abstract understanding. If pre-service teachers are able to think from concrete to abstract or vice versa abstract to the concrete, it means that the concept can be thoroughly and students can already be brought to other higher topics. These three stages in CRA support each other and their implementation is not linear but cyclic. In addition, it is also based on the results of research that using concrete objects is more developed and more comprehensive in making representations and showing good motivation in the habit of doing tasks, understanding mathematical ideas, and better applying mathematical ideas to everyday life, studies have concluded that there is an effect of retention and self-efficacy using CRA compared to those using traditional learning (Harrison & Harrison, 1986; Hughes, 2011). Some of the mathematical concepts described using concrete materials are very good as a foundation or basis in developing the ability to understand concepts about the relationship between numbers, places and values, counting, fractions, decimals, measurements, geometry, money, percentages, base numbers, statistics, and probabilities.

Learning with the CRA approach provides an opportunity for students to explore things observed during the learning process, then compare them with things that are already known. Students construct their own knowledge, while lecturers guide and help if pre-service teachers find difficulties or make mistakes. The role in the CRA learning process is required to use the knowledge and abilities of pre-service teachers themselves in recognizing, understanding, solving problems or material presented by lecturers. Pre-service teachers inevitably have to believe in their abilities, so can judge for themselves, consider themselves to have a role, in other words, they feel valuable or worthy to play a role in solving problems encountered during the CRA learning process. The CRA approach has been investigated by several researchers who stated that CRA was successfully applied in learning (from small groups to classical) and was very useful when learning to use this approach to significantly get higher scores from their peers who were taught traditionally and had a positive impact from the CRA approach to the ability to understand concepts and problem solving abilities (Arvianto, 2011; Hughes, 2011; Witzel et al., 2008).

Related to the assessment, it is necessary to have alternative assessments that can improve the ability of students and allow lecturers to assess each assignment given to pre-service teachers and provide feedback so that they can be seen achieving improvement in abilities, as well as weaknesses to be improved. In relation to the achievement of the desired competencies, it is necessary to have an assessment carried out with a continuous process so that pre-service teachers competency achievement can be seen. This means that the process experienced must pay attention to their judgment compared to just paying attention to the end result. It is a logical fact that getting good output should start with a good thing. Therefore it is necessary to evaluate alternatives aimed at the process and learning outcomes of pre-service teachers. Alternative assessment means non-traditional assessment formats, usually requiring the construction, demonstration, or performance. Authentic, an alternative format is done specifically and student-centered. An alternative assessment that has characteristics like this is a portfolio. Portfolio assessment provides an overview of the progress of learning of students seen from their output/work (Uno & Koni, 2014).

Class-based portfolio assessment and see the collection of works of pre-service teachers who are systematically organized and organized in a certain time span (Surapranata & Hatta, 2004). Portfolios are one of the developing approaches used to monitor various forms of pre-service teachers work. Various studies using portfolio assessments have been carried out concluded that learning using portfolio assessment will obtain higher learning outcomes than those who do not use portfolio assessment by controlling numerical talent (Setemen, 2014). Portfolio assessment is in line with the concept of competency-based assessment, that authentic assessment (such as a portfolio) is appropriate because it will provide a continuous assessment process for pre-service teachers to achieve their competencies. But if it is associated with differences in the characteristics of intelligence of each individual, it is necessary to do an individual study of the suitability of portfolio assessment in improving pre-service teachers logical thinking ability. As a follow-up, the researcher wishes to know and examine the development of logical thinking ability through the CRA (Concrete-Representational-Abstract) approach with portfolio assessment.

METHOD

This research is quantitative research in education that is research using pretest before learning and given posttest after learning in class (pretest-posttest group design). Giving pretest and posttest is intended to see the comparison of differences in abilities before and after learning. This research is a quasi-experimental with time series design. The design used one group time series design or simple interrupted time series (Campbell & Stanley, 1963, 1966). with the following forms: O_1 O_2 O_3 O_4 X O_5 O_6 O_7 O_8, O_1-O_4 is measurements or tests of mathematical logical thinking abilities before learning, X is a learning treatment with a CRA approach with portfolio assessment, O_5-O_8 is measurements or tests of mathematical logical thinking abilities after learning. The mathematical logical thinking ability test instruments used consists of 4 packages in this case (paired) between pretest1 (O_1) with posttest1 (O_5) using package 1, pretest2 (O_2) with posttest2 (O_6) using package 2, and so on.

Subjects and Treatments

The subject of this study consist of 37 first years pre-service teachers of mathematics education in one of the universities in Banten province, Indonesia. The pre-service teachers in the study sample at the beginning of the implementation were given a pretest 4 times a week to see patterns or trends of logical thinking ability before treatments, then the CRA approach treatments with portfolio assessment was carried out in 5 weeks with the material and the subject according to the lecture, after completing the treatment, the posttest was measured using the same instrument 4 times as well to see the pattern of trends of the ability to think logically after the treatment.

DISCUSSION

Mathematical-logical thinking ability overall data on pretest 1, posttest 1, pretest2, posttest2, pretest3, posttest3, pretest4 and posttest4 can be seen in the following table:

Table 1. Descriptive Statistics Mathematical Logical Thinking Ability

	Mean	Median	Mode	Std Deviation	Variance	Minimum	Maximum
Pretest1(O_1)	-0.30	-0.21	-0.72	0.33	0.11	-0.86	0.31
Posttest1 (O_5)	1.58	1.51	1.51	0.40	0.16	0.97	2.47
Pretest2 (O_2)	-0.36	-0.28	-0.61	0.36	0.11	-1.22	0.31
Posttest2 (O_6)	1.15	1.21	1.40	0.25	0.06	0.64	1.40
Pretest3 (O_3)	-0.30	-0.21	-0.08	0.49	0.24	-1.73	0.44
Posttest3 (O_7)	2.29	2.29	2.29	0.50	0.25	1.31	3.34
Pretest4 (O_4)	-0.25	-0.20	-0.49	0.39	0.15	-1.29	0.51
Posttest4 (O_8)	3.03	3.11	4.39	1.13	1.27	0.92	4,39

n = 37 subjects

From the table above, it gives an overview of the average, median, mode values, for each pretest and posttest that is at pretest1 the average initial ability of mathematical logical thinking -0.30 logit, there were 57% (21 pre-service teachers) have initial abilities ≥ - 0.30 logit and 43% (16 pre-service teachers) have initial abilities < -0.30 logit. At the pretest2, the initial ability of mathematical logical thinking -0.36 logit, there were 51% (19 pre-service teachers) who had the initial ability ≥ -0.36 logit and 49% (18 pre-service teachers) had initial abilities < -0.36 logit. At pretest3 the average initial ability of mathematical logical thinking -0.30 logit, there were 57% (21 pre-service teachers) who had the same initial ability ≥ -0.30 logit and 43% (16 pre-service teachers) who had initial abilities < -0, 30 logit. While at pretest4 the average initial ability of mathematical logical thinking -0.25 logit, there were 51% (19 pre-service teachers) who had the initial ability ≥ -0.25 logit and 49% (18 pre-service teachers) had initial abilities < -0.25. Whereas for each posttest data that is in posttest1, the average final ability of mathematical logical thinking is 1.58 logit, there are 43% (16 pre-service teachers) have final abilities ≥ 1.58 logit and 57% (21 pre-service teachers) have final abilities < 1.58 logit. In posttest2 the average final ability of mathematical logical thinking was 1.15 logit, there were 51% (19 pre-service teachers) of students who had the final ability of ≥ 1.15 logit and 49% (18 pre-service teachers) had the final ability < 1.15 logit. In posttest3 the average final ability of mathematical logical thinking was 2.29 logit, there were 68% (25 pre-service teachers) had the final ability of 2.29 logit and 32% (12 pre-service teachers) who had the final ability < 2.29 logit. While in posttest4 the average final ability of mathematical logical thinking was 3.03 logit, there were 54% (20 pre-service teachers) who had the final ability of ≥ 3.03 logit and 46% (17 pre-service teachers) had the final ability < 3.03 logit. Furthermore, inferential statistical analysis is carried out to test the research hypothesis with the average two-parameter difference test technique or paired sample t-test, obtained:

Table 2. Paired Sample T-Test Pretest-Posttest

		Mean	Std. Deviation	Std. Error Mean	t	df	p-value
Pair 1	(O_1) - (O_5)	1.88	0.55	0.09	20.76	36	0.000
Pair 2	(O_2) - (O_6)	1.51	0.42	0.07	21.96	36	0.00
Pair 3	(O_3) - (O_7)	2.59	0.74	0.12	21.27	36	0.00
Pair 4	(O_4) - (O_8)	3.28	1.11	0.18	17.98	36	0.00

The average mathematical logical thinking ability of pre-service teachers in the first test before treatment (O_1) mean -0.30 logit, standard deviation 0.33 logit and the first test after treatment (O_5) mean 1.58 logit, standard deviation 0,42 logit. This is descriptively the ability of mathematical logical thinking of pre-service teachers in the first test after treatment with the CRA approach is higher than the first test before CRA. Based on paired sample t-test t_{count} = 20.76 for df = 36 and α = 0.05 obtained $t_{table} = t_{(0.05;\ 36)} = 2.028$. It shows that $t_{count} > t_{table}$ or because the p-value = 0,000 < 0.05 then H_0 is rejected, which means the mathematical logical thinking ability of pre-service teachers in the first test after treatment (O_5) is significantly higher than the first test before treatment (O_1) with using the CRA approach with portfolio assessments. On the second test before treatment (O_2) mean -0.36 logit, standard deviation was 0.34 logit and the second test after treatment (O_6) was 1.15 logit, the standard deviation was 0.25 logit. This is descriptively mathematical logical thinking ability of pre-service teachers in the second test after treatment with the CRA approach is higher than the second test before treatment. Based on paired sample t-test t_{count} = 21.96 for df = 36 and α = 0.05 obtained $t_{table} = t_{(0.05;\ 36)} = 2.028$. It shows that $t_{count} > t_{table}$ or because the p-value = 0.000 < 0.05 then H_0 is rejected, which means that pre-service teachers logical thinking ability in the second test after treatment (O_6) is significantly higher than the second test before treatment (O_2) using the CRA approach with portfolio assessments.

On the third test before treatment (O_3) mean -0.30 logit, standard deviation 0.49 logit and the third test after treatment (O_7) mean 2.29 logit, standard deviation 0.50 logit. This is descriptively mathematical logical thinking ability of pre-service teachers on the third test after treatment with the CRA approach is higher than the third test before treatment. Based on paired sample t-test t_{count} = 21.27 for df = 36 and α = 0.05 obtained $t_{table} = t_{(0.05;\ 36)} = 2.028$. It shows that $t_{count} > t_{table}$ or because the p-value = 0.000 < 0.05 then H_0 is rejected, which means that pre-service teachers logical thinking ability in the third test after treatment (O_7) is significantly higher than the third test before treatment (O_3) using the CRA approach with portfolio assessments. On the fourth before treatment (O_4) mean -0.25 logit, standard deviation 0.39 logit and test four after treatment (O_8) mean 3.03 logit, standard deviation 1.13 logit. This is descriptively mathematical logical thinking ability of pre-service teachers on the fourth test after treatment with the CRA approach is higher than the fourth test before treatment. Based on paired sample t-test t_{count} = 17.98 for df = 36 and α = 0.05 obtained $t_{table} = t_{(0.05;\ 36)} = 2.028$. It shows that $t_{count} > t_{table}$ or because the p-value = 0,000 < 0.05, H_0 is rejected, which means that pre-service teachers logical thinking ability in the fourth test after treatment (O_8) is significantly higher than the fourth test before treatment (O_4) using the CRA approach with portfolio assessments.

Then, compared to the overall mathematical logical thinking ability of pre-service teachers after learning (O_{10}) using the CRA approach with portfolio assessments compared to before treatment (O_9) is obtained:

Table 3. Results of Comparison of Posttes (O_{10}) Pretest (O_9)

Statistics	Pretest (O_9)	Posttes (O_{10})
Mean	-0.301	2.015
Standard Deviasi	0.212	0.342
Std. Error Mean	0.035	0.056
N (subjects)	37	

Table 4. The t-test (independent sample test) Average Posttest (O_{10}) Pretest (O_9)

Levene's Test for Equality of Variances		*t-test for Equality of Means*		
F	Sig.	t	df	Sig. (2-tailed)
13.292	0.001	35.037	72	0.000

Based on the table mathematical logical thinking ability of pre-service teachers before treatment Pretest (O_9) mean -0.301 logit, standard deviation 0.212 logit and after treatment Posttest (O_{10}) mean 2,015 logit, standard deviation 0.342 logit. This is descriptively mathematical logical thinking ability of pre-service teachers after treatment with the CRA approach is higher than before treatment. At Levene's test for Equality of Variance, the statistical F = 13.292 with sig. or p-value = 0.001 < 0.05, it shows that the population variance of the two groups is not homogeneous or heterogeneous. In the t-test for Equality of Means, $t_{count} = 35.037$. Based on (df) = n1 + n2 - 2 = 72 with $\alpha = 0.05$ obtained $t_{table} = t_{(0.05;\ 72)} = 1.9935$. This means $t_{count} > t_{table}$ or because the p-value = 0.000 / 2 = 0.000 < 0.05, H_0 is rejected, it shows that pre-service teachers logical thinking ability after treatment (O_{10}) is significantly higher than before treatment (O_9) using the approach CRA with portfolio assessments.

Trends of Mathematical Logical Thinking Ability

The trends of mathematical logical thinking ability pre-service teachers before the CRA approach with a portfolio assessment if the graphic forms are as follows:

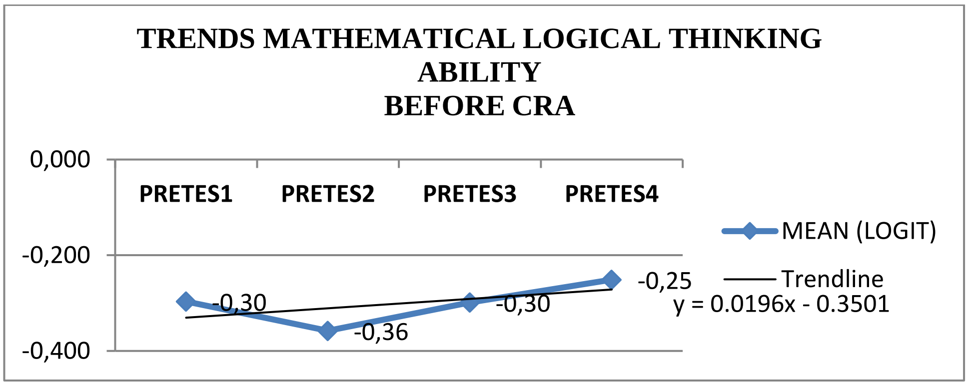

Figure 1. Trends of Mathematical Logical Thinking Ability before CRA Approach with Portfolio Assessment

From Figure 1. the development of the average mathematical logical thinking ability has not been influenced by the CRA approach with portfolio assessment. Trendline mathematical logical thinking ability before CRA with equation y = 0.0196x -0.3501. If seen the development trends of mathematical logical thinking ability before learning between mathematical logical thinking ability in the first pretest (O_1), the second pretest (O_2), the third pretest (O_3) and the fourth pretest (O_4) obtained mean of -0.30; -0.36; -0.30 and -0.25 logit, then to see the difference between the trends (development) of mathematical logical thinking ability before treatment, a comparative analysis using the F test is:

Table 5. Levene's Test of Equality of Error Variances Before CRA

F	df1	df2	Sig.
1.633	3	144	.184

The hypothesis is H_0: $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$; H_1: other than H_0. Based on the table above the Levene's test of error variance is expressed with statistics F = 1.633 with df1 = 3, df2 = 144, or p-value = 0.184 > 0.05 then H_0 is accepted. So that the average parameters of the four tests before treatment have the same or homogeneous variance.

Table 6. One Way ANOVA Trends Before CRA

Source	Type I Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.213[a]	3	.071	.461	.710
Intercept	13.416	1	13.416	87.216	.000
A	.213	3	.071	.461	.710
Error	22.151	144	.154		
Total	35.780	148			
Corrected Total	22.364	147			

a. R Squared = .010 (Adjusted R Squared = -.011)

The hypothesis is H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: other than H_0. Based on the analysis obtained F_{count} = 0.461 while $F_{table}(0.05; 3.144)$ = F_{table} = 2.667. It shows that $F_{count} < F_{table}$ or with p-value = 0.710 > 0.05 so H_0 is accepted, it can be concluded that in the time sequence before treatment with the CRA approach there was no difference in pre-service teachers mathematical logical thinking ability. While the trends of mathematical logical thinking ability of pre-service teachers after CRA with portfolio assessment graphs are as follows:

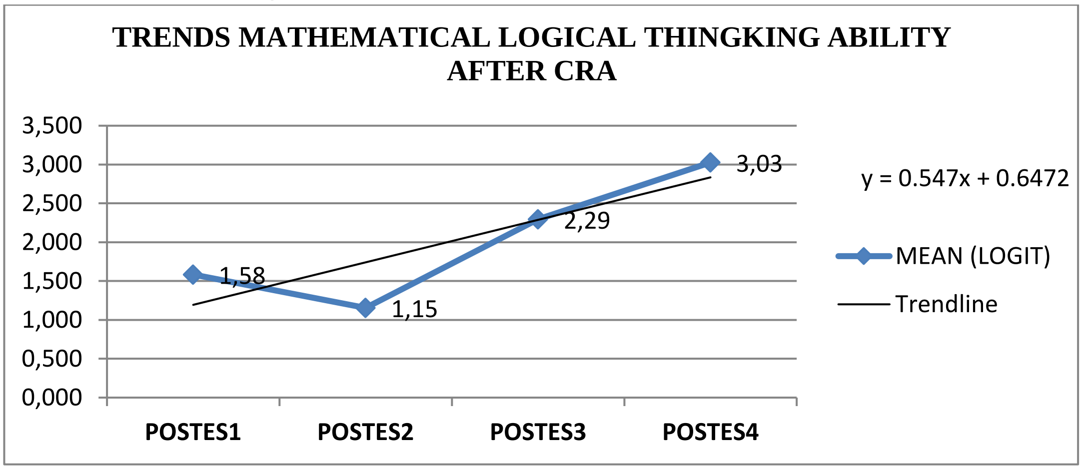

Figure 2. Trends of Mathematical Logical Thinking Ability After CRA Approach with Portfolio Assessment

From Figure 2. above, it can be seen that trendline equation y = 0.547x + 0.6472, shows that the contribution given after CRA with portfolio assessment of mathematical logical thinking ability of 0.547 or the average increase (or decrease) in thinking ability logically mathematical equal to 0.547 for each increase in one unit. So that from the trendline before and after CRA is obtained: 1) changes in the level of ability 0.9973 and 2) changes in slope from 0.0196 to 0.547.

If seen the development trends of mathematical logical thinking ability after learning between mathematical logical thinking ability in the first posttest (Q_5), the second posttest (Q_6), the third posttest (Q_7) and the fourth posttest (Q_8) obtained mean of 1.58; 1.15; 2.29 and 3.03 logit, so to see the difference between mathematical logical thinking ability after learning a comparative analysis using the F test can be done in:

Table 7. Levene's Test of Equality of Error Variances After CRA

F	df1	df2	Sig.
39.284	3	144	0.000

Based on the table above the Levene's test of error variances is stated with statistics F = 39.284 with df1 = 3, df2 = 144, or p-value = 0.000 < 0.05, so H_0 is rejected. So that the average parameters of the four tests after CRA have a variance not equal or heterogeneous

Table 8. One way ANOVA Trends After CRA

Source	Type I Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	75.016[a]	3	25.005	57.272	.000
Intercept	600.672	1	600.672	1375.775	.000
B	75.016	3	25.005	57.272	.000
Error	62.871	144	.437		
Total	738.559	148			
Corrected Total	137.887	147			

a. R Squared = .544 (Adjusted R Squared = .535)

The hypothesis is H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: Other than H_0. Based on the analysis obtained F_{count} = 57.272 while F_{table} (0.05; 3.144) = F_{table} = 2.667. It shows $F_{count} > F_{table}$ or with p-value = 0.000 < 0.05 so H_0 is rejected, which means that it can be concluded that in the time sequence after CRA approach there are differences in pre-service teachers mathematical logical thinking ability. So, overall the development or trends of mathematical logical thinking ability of pre-service teachers before and after the CRA approach with portfolio assessment can be seen in the following graph:

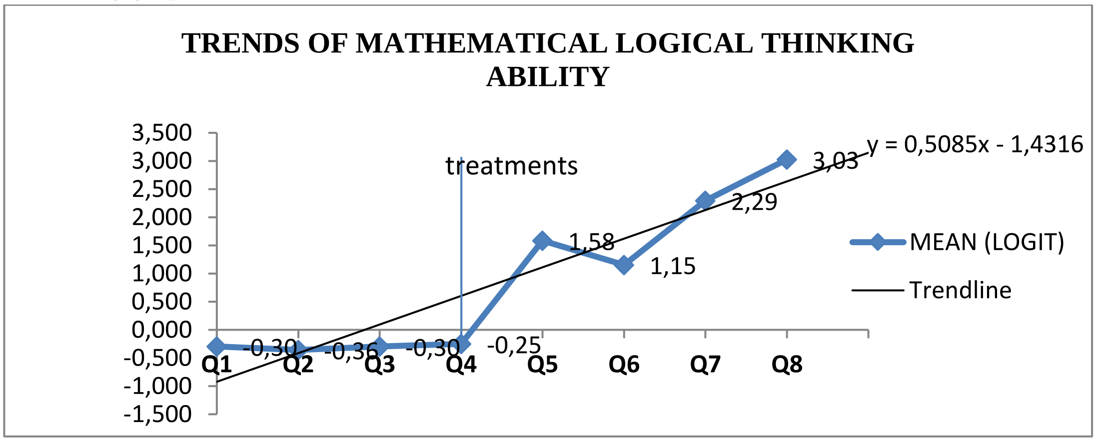

Figure 3. Trends of Mathematical Logical Thinking Ability Before and After CRA Approach with Portfolio Assessment

The left chart illustrates the ability of students before CRA with portfolio assessment (O_1-O_4), and treatments line, then the right chart illustrates the ability of pre-service teachers after CRA with portfolio assessment (O_5-O_8). It is apparent that the graph after treatment rises from the side of the logit value, and if seen at the graph pattern between the graph to the left of the treatments line and the right of the treatment the pattern is the same and increases, this shows good results after learning using the Concrete-Representational-Abstract (CRA) approach with portfolio assessment of individual or pre-service teachers development patterns according to the mathematical logical thinking ability of pre-service teachers through CRA with portfolio assessment develops and changes occur optimally. The development of logit values of pre-service teachers mathematical logical thinking ability before CRA with portfolio assessment -0.30; -0.36; -0.30; and -0.25 while for the development of logit values the ability of pre-service teachers to think logically after CRA with portfolio assessment of +1.58; +1,15; +2,29; and +3.03. If in relation to trendline the development of pre-service teachers mathematical logical thinking abilities tends to rise or increase above the trendline equation $y = 0.5085x - 1.4316$. From this equation, it means the slope of the line is 0.5085 where the slope is the vertical distance divided by the horizontal distance between the two points on the line, which is the rate of change along the regression line. The slope is a regression coefficient for variable x (independent variable), slope is a value that shows how much contribution is given a variable x to y, the value of slope can be interpreted as the average increase (or reduction) that occurs in variable y for each increase in one unit of variable x. So that it can be said that the contribution or the average increase (or reduction) given by CRA approach with portfolio assessment of mathematical logical thinking ability of 0.5085.

CONCLUSION

So that it can be concluded 1) after using the CRA approach with portfolio assessment of pre-service teachers mathematical logical thinking ability in the first test after learning (O_5) is higher than the first test before learning (O_1), 2) After using the CRA approach with portfolio assessment of pre-service teachers mathematical logical thinking ability in the second test after learning (O_6) are higher than the second test before learning (O_2). 3) After using the CRA approach with portfolio assessment of pre-service teachers mathematical logical thinking ability in the third test after learning (O_7) is higher than the third test before learning (O_3). 4) After using the CRA approach with portfolio assessment of pre-service teachers mathematical logical thinking ability on the fourth test after learning (O_8) is higher than the fourth test before learning (O_4). 5) Overall the ability of mathematical logical thinking after used CRA (O_{10}) approach with portfolio assessment is higher than the ability of mathematical logical thinking before CRA (O_9). 6) The development of pre-service teachers mathematical logical thinking ability after CRA has trends to increase above the trendline with the equation $y = 0.5085x - 1.4316$. So that it can be said that the contribution or an average increase (or reduction) given by CRA approach with portfolio assessment of mathematical logical thinking ability of 0.5085.

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