SOME DENSITY PROFILES OF AN INHOMOGENEOUS SCHRÖDINGER EQUATION

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Abstrak

Telah didapatkan profil kerapatan dari persamaan Schrödinger non homogen dari beberapa fungsi yang merepresentasikan suku non homogen. Persamaan Schrödinger non homogen ini, yang didapat dengan mereduksi sebuah set dari dua persamaan Gross-Pitaevskii yang terkopel, menggambarkan sebuah persamaan gerak dari sebuah laser atom yang terkopel lemah di dalam daerah kondensat. Dalam hal ini, profil kerapatan didapat melalui metode separasi variabel untuk mempermudah perhitungan.

Abstract

We provide some density profiles of an inhomogeneous Schrödinger equation by giving some available functions representing the inhomogeneous term. The inhomogeneous Schrödinger equation, which is achieved by reducing a set of two-coupled Gross-Pitaevskii equations, describes a motion equation of a weakly outcoupled atom laser inside the condensate zone. In this case, to attain the density profile we use the separation of variables in order to simplify our calculation.

Keywords: *density profile, atom laser, Schrödinger equation.*

1. Introduction

It is briefly noted that the year of 1995 was considered as a historical year because the existence of the Bose-Einstein condensation has been verified by several experiments in evaporatively cooling alkali atoms at very low temperatures, minimally near the appropriate critical temperature, for example see [1-3]. The physicists confirmed the existence by observing the sharp peak of both coordinate distribution and momentum distribution which is shown that de Broglie's wavelengths of atoms are clearly overlapped each other [4]. It means that all atoms now occupy the lowest state, namely the gound state, forming a Bose-Einstein condensate. In the available experiments, all atoms are initially placed in the magnetic confinement generated by the electromagnetic source, so in the quantum language we can say the atoms are in the trapped state.

An atom laser is considered as an extracted output produced by flipping the confined atoms from the trapped state to the untrapped state [5-8]. Those atoms, which pose alkali atoms in the Bose-Einstein condensation experiments, are initially confined in the anisotropic trap which is generated by the magnetic field until reaching the appropriate critical temperature. To flip the trapped atoms, one must apply an outcoupler, such as a radio frequency, that is used by the atoms as an energy source to leave the confinement.

In fact, the extracted atom laser experiences two zones, namely, the zone that atom laser is inside the Bose-Einstein condensate region and outside the Bose-Einstein condensate region [9]. Those two regions are determined by the appropriate coupling term as a source term. In this case, both the dynamics of condensate and atom laser is governed by a set of two coupled three-dimensional Gross-Pitaevskii equations. If the assumption that the extracted atom laser is sufficiently diluted, the appropriate Gross-Pitaevskii equation representing the atom laser motion will be reduced to Schrodinger equation, namely, the homogeneous Schrodinger equation for inside the condensate region and the inhomogeneous Schrodinger equation for outside the condensate region [9].

In this paper, we provide some density profiles of the atom laser beam wave function inside the Bose-Einstein condensate. Actually, in our previous paper we have provided one density profile whose the source term is represented as a cosine function [10]. The profile describes a damped-like oscillation and only defines in the certain interval. Moreover, the solution possibilites outside the interval are not available because can not be defined both analytically and numerically. Therefore, our motivation is to continue our calculation by employing the different source functions. The paper will be organized as follows. In the second section we briefly review the appropriate differential equations to construct the wave function of the atom laser by applying the dilute condensate. Later, in the third section we attach some density profiles related to the appropriate differential equation of the dynamics of atom laser beam. We give our conclusions at the end of the section.

2. Method

In this section we briefly discuss all of the available differential equations governing both the condensate motion and atom laser motion. Firstly, the differential equations representing the dynamics of the condensate and the weakly outcoupled atom laser inside the condensate region are given respectively as follows [9-11]

$$i\hbar\frac{\partial\psi_c}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V_c+g_{cc}|\psi_c|^2\right)\psi_c \quad (1)$$

$$i\hbar\frac{\partial\psi_\ell}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V_\ell+g_{c\ell}|\psi_c|^2\right)\psi_\ell+\chi \quad (2)$$

In this case, ψ_c and ψ_ℓ are the wave functions of condensate and atom laser respectively, V_c and V_ℓ represents the external potentials

$$V_c(\vec{r})=\frac{1}{2}m\omega^2\left(x^2+y^2+\lambda_z{}^2z^2\right) \quad (3)$$

$$V_\ell(\vec{r})=-\mu+\frac{1}{2}m\left[\omega_\perp^2(x^2+y^2)+\omega_z^2z^2\right]+\frac{1}{2}m\omega_\perp^2\sigma^2 \quad (4)$$

where g_{cc} and $g_{c\ell}$ are the strength interaction of states, and χ describes the source term. For the important information, if we set χ as zero, Eq. (2) leads to the atom laser motion outside the condensate region whose solution accepts a linear superposition for the certain case [12].

If we set the cigar-shaped trap in Eq. (2), the solution of the condensate motion can be given in the cylindrical coordinates as [13]

$$\psi_c(\rho,z,t)=\sqrt{\frac{N}{a_0^3}}\lambda_z^{1/8}\pi^{-3/4}\exp\left(-\rho^2/2-z^2/2\right)\exp\left(i[\kappa+\omega_\rho]t\right) \quad (5)$$

By using the following transformations

$$(x,y,z)\rightarrow(x,y,z)/a_0 \quad (6)$$

$$t\rightarrow t/2\omega_z \quad (7)$$

$$\psi_\ell\rightarrow\psi_\ell/\sqrt{\hbar\omega_\perp/2\sqrt{\lambda_z}} \quad (8)$$

where a_0, ω_z, and $\omega_\perp$ have usual definitions, and redefining the following functions

$$\psi_\ell=\alpha(\rho)\beta(z)e^{-i\gamma t} \quad (9)$$

$$\chi=\alpha(\rho)\phi(z)e^{-i\gamma t} \quad (10)$$

we obtain the differential equation of the longitudinal profile of the atom laser

$$\frac{d^2\beta}{dz^2}-z^2\beta-g_{c\ell}|\psi_c|^2\beta+\Omega\beta=-\phi \quad (11)$$

where we write

$$\Omega=\frac{1}{2}m\omega_\perp^2\sigma^2/\sqrt{\lambda_z}-\mu/\sqrt{\lambda_z} \quad (12)$$

For the complete explanation how to find the above equation, the readers are welcome to see [10].

3. Discussion

Observing Eq. (11), we finally multiply $|\psi_c|^2$ both sides in Eq. (11), integrating over the radial part ρ, and assuming that

$$e^{-2z^2}\approx 1-2z^2 \quad (13)$$

we achieve the longitudinal propagation of the atom laser

$$\frac{d^2\beta}{dz^2}+(2g_{c\ell}-1)z^2\beta+(\Omega-g_{c\ell})\beta=-\phi \quad (14)$$

Since the analytical solution is not available we attach below the numerical solutions for some cases by inputting ϕ.

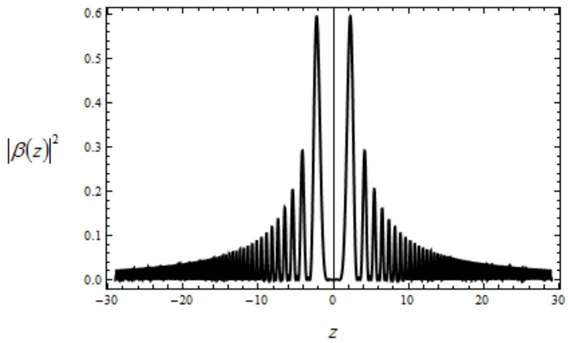

Fig 1. Density longitudinal profile of the weakly outcoupled atom laser for $\phi=z$

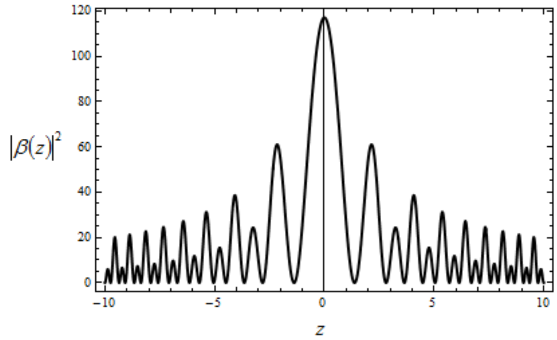

Fig 2. Density longitudinal profile of the weakly outcoupled atom laser for $\phi=z^2$

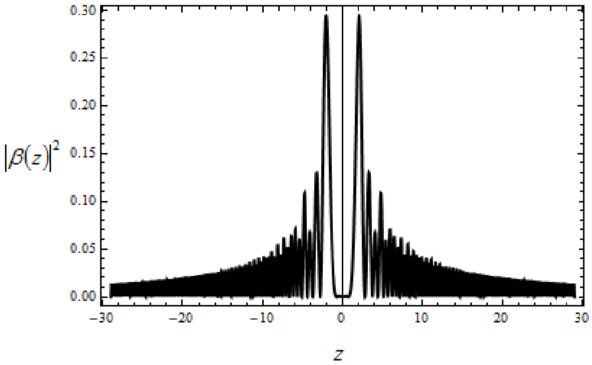

Fig 3. Density longitudinal profile of the weakly outcoupled atom laser for $\phi = \sin z$

The figures above show the localized solutions only occur in the certain interval and outside the interval the solutions are not available.

4. Conclusion

The nonlinear differential equation, namely, a set of two coupled three dimensional Gross-Pitaevskii equations governing the condensate and atom laser motions can be decoupled into two separated differential equations representing each motion if the atom laser itself is sufficiently diluted. In this case the interaction between atoms is neglected. However, we also need the solution of the ordinary Gross-Pitaevskii equation governing the condensate motion that must be substituted to the Schrodinger equation representing the atom laser.

By using the separation of variables we attach some figures attained from numerical solutions for some cases. We see that all figures only defined in the certain intervals and all the solutions are localized. If we want to redefine the interval to infinity, we have to rescale the interval.

Acknowledgement

We thank all colleagues for the motivation so that the paper can be published.

Appendix

A set of two coupled Gross-Pitaevskii equation that includes both a condensate and an atom laser can be written as

$$i\hbar \frac{\partial \psi_i}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_i(\vec{r}) + \sum_{k=\ell,c} g_{ki}|\psi_k|^2 \right)\psi_i + W_{ij}(\vec{r},t)\psi_j \qquad (15)$$

where i is a running index that runs from condensate index to atom laser index. All of the detail explanations of the each symbol can be found in [9].

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p-ISSN: 2339-0654
e-ISSN: 2476-9398