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Pair Correlation Influence on Superconductors Josephson Penetration Depth

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ABSTRACT

The Josephson penetration depth is an essential characteristic of Josephson junctions, serving a role akin to the London penetration depth in bulk superconductors. It originates from the substantial self-magnetic field generated by a strong Josephson supercurrent, influencing the distribution of the gauge invariant phase difference across the junction. This study delves into the intricate relationship between cooper pair correlation and critical temperature in superconductors. To study relationships authors develop theoretical method and observed that critical temperature exhibits a noteworthy decrease with an increase in cooper pair correlation. Specifically, as the level of coherence among electron pairs rises, the material's capacity to maintain the superconducting state at elevated temperatures is enhanced, resulting in an elevated critical temperature. Conversely, regions characterized by lower pair correlation demonstrate a sharp reduction in critical temperature, indicating their heightened susceptibility to changes in correlation levels. This sensitivity is particularly pronounced across junction and penetration depth where cooper pair correlation is diminished. Furthermore, the study reveals an exponential decay trend in critical temperature concerning cooper pair correlation, underscoring the pivotal role played by pair correlation in the superconducting state. Even slight alterations in pair correlation have a substantial impact on the material's ability to exhibit superconductivity. These findings provide valuable insights for the tailored design and optimization of superconducting materials for specific applications. By leveraging the understanding gained from this research, it becomes possible to engineer materials with enhanced superconducting properties. This study not only advances our fundamental comprehension of superconductivity but also offers practical implications for a diverse range of technological applications.

Keywords: superconductors, critical temperature, cooper pair correlation, coherence

INTRODUCTION

Superconductivity emerges when a large number of conduction electrons in a metal form Cooper pairs at extremely low temperatures. This transition represents an order-disorder phase change, and the dimensionality of the system (1D, 2D, or 3D) plays a crucial role in determining its properties. In lower dimensions, such as 2D, the phase transition becomes more challenging because the spatial confinement limits interactions between microscopic components (e.g., electrons), resulting in fewer interaction partners. Consequently, even below the transition temperature (Tc), different parts of the system tend to behave more independently, leading to greater spatial and temporal fluctuations in the order parameter. The analysis of superconducting states, specifically focusing on their wave functions. The utilizes the Bogoliubov-de Gennes (BdG) equation, with an emphasis on s-wave pairing interaction. The comparative impacts of paramagnetic and diamagnetic pair-breakings on key parameters, such as the pair potential, screening current, spin current, and paramagnetic moment. The depth-dependent behavior of crucial quantities, including the pair potential, supercurrent, and spin current [1].

Superconductivity exist in a 2D system, particularly when one dimension is reduced to atomic-scale size. The unique characteristics and phenomena can expect in 2D superconductivity. Historically, it's worth noting that the Mermin-Wagner theory prohibits the traditional superconducting phase transition with long-range order parameter correlation in 2D systems. The Kosterlitz-Thouless-Berezinskii (KTB) transition, consistent with the Mermin-Wagner theory, allows quasi-long-range correlation of the order parameter. Even without the KTB transition, Cooper pairs can condense at the mean-field level due to the Bardeen-Cooper-Schrieffer (BCS) mechanism. In practice, a system may be considered superconducting if the order parameter's correlation is sufficiently developed at low temperatures [2]. The propagation of pair correlation from a superconductor-normal-metal (SN) interface into the normal metal has been explored using STM/STS. Near the interface in the normal metal, the single-particle spectrum displays a dip at the Fermi energy (EF), a measure of superconducting pair correlation [3]. The findings indicate that pair correlation and Josephson penetration depth are not directly linked.

Theoretical proposals have suggested that in two-dimensional (2D) superconductors with strong spin-orbit coupling and an in-plane magnetic field, Cooper pairs could acquire finite momentum, leading to this diode effect. The researchers discovered a giant Josephson diode effect in Josephson junctions made from a type-II Dirac semimetal, $NiTe_2$. They found that the asymmetry in the critical current depends on the magnitude and direction of an applied magnetic field, achieving its maximum value when the field is perpendicular to the current. The observed characteristics were explained by a model based on finite-momentum Cooper pairing, mainly originating from the Zeeman shift of spin-helical topological surface states [4]. The research gap here lies in the need for further studies to explore and confirm the link between finite-momentum Cooper pairing and the Josephson penetration depth. This connection could provide deeper insights into the observed diode effect and its potential applications in quantum technologies.

Pair-density-wave (PDW) superconductivity is another exotic state characterized by an oscillating superconducting order parameter, and it doesn't require an external magnetic field. Establishing a two-dimensional (2D) microscopic model with PDW order in its ground state has been rare. In a recent study, a minimal model of spinless fermions on a honeycomb lattice with nearest-neighbor (NN) and next-nearest-neighbor (NNN) interactions was explored [5]. Exploring the intersection of magnetism and superconductivity is vital for uncovering unconventional and potentially topological superconductors. The 2D metal with d-wave altermagnetism and Rashba spin-orbit coupling. It reveals that the system predominantly exhibits a mix of spin-singlet s-wave and spin-triplet p-wave pairings, due to the absence of time-reversal and inversion symmetries. Altermagnetic metals offer intriguing insights into intrinsic unconventional and topological superconductivity [6].

The study of high-temperature superconductors, focusing on the 2D Hubbard model, a fundamental model for strongly correlated electron systems. The research reveals that d-wave pairing correlation slightly increases when the positive on-site Coulomb interaction is present and doping occurs away from half-filling. The increase is proportionate to system size, suggesting the presence of a superconducting phase. However, no enhancement is observed at half-filling, signifying the absence of superconductivity without hole doping [7].Understanding the microscopic mechanism behind high-temperature superconductivity, particularly in copper-oxide materials, has been a long-standing challenge. While the d-wave symmetry of the order parameter and proximity to an antiferromagnetic Mott phase suggest strong correlation effects are at play, identifying the exact "pairing glue" has proven elusive. To address this, a focused study investigates the origin of superconductivity in the 2D single-band Hubbard model, a minimal theoretical representation of cuprate superconductivity. Spin fluctuations have been a prominent explanation in weak-coupling regimes, but it's unclear how this applies to cuprate materials with stronger interactions. To resolve this, the study employs fluctuation diagnostics, treating various fluctuations equally to identify those driving anomalous self-energy in the superconducting state, bridging a gap in our understanding of high-temperature superconductivity.

After the discovery of high-temperature cuprate superconductors, the 2D Hubbard model became central to understanding superconductivity. It was believed that strong antiferromagnetic fluctuations, arising from the interplay between electron hopping and on-site repulsion, played a key role [8]. High-temperature superconducting cuprates exhibit a complex interplay of striped magnetic and charge orders alongside superconductivity. Surprisingly, similar behavior has emerged in numerical simulations of the Hubbard model, which represents the copper-oxygen layers in these materials. The investigation of their impact on the model's superconducting properties, revealing evidence of pair-density-wave correlations intertwined with stripe correlations [9]. The practical application of influence of cooper pairs correlation was used in superconducting quantum interference devices (SQUIDs) and magnetic resonance imaging (MRI).

The fluctuation diagnostics method to the superconducting phase and unequivocally identified spin fluctuations as the dominant contributor to d-wave pairing in the Hubbard model, even at interaction strengths relevant to cuprates, beyond the weak-coupling regime. However, no evidence supporting alternative scenarios like nematic fluctuations, loop current order, or intertwined orders. The relationship between Cooper pairs and Josephson penetration depth, leaving this as a potential research gap was not studied in their work [10-11]. The identification of superconducting mechanisms aligns with experimental evidence. Studies show agreement between spectral functions from spin fluctuation theory and inelastic neutron scattering, as well as angle-resolved photoemission spectroscopy in $YBa_2Cu_3O_{6.66}$. However, experiments suggesting links between superconductivity and charge density wave or differences between pseudogap and superconductivity origins may be probing aspects beyond the single-orbital Hubbard model on an eight-site DCA cluster [12-13].

The identification of superconducting mechanisms, specifically spin fluctuations, is consistent with diverse experimental evidence. However, conventional spin fluctuation theory often underestimates or overestimates pairing contributions due to its limitations beyond the weak-coupling regime. The fluctuation diagnostics precisely pinpoint antiferromagnetic spin fluctuations as the driving force behind d-wave pairing in the Hubbard model, particularly for intermediate-to-large electronic interactions relevant to cuprate physics [14]. Many-body systems display non-equilibrium physics, in which the constituent particles interact intensely and behave quantum mechanically. The study of non-equilibrium processes has been extended to the quantum domain thanks to recent technological developments in mesoscopic and atomic systems. External disturbances such as laser irradiation, microwave irradiation, phonon injection, and quasiparticle injection via tunnel junctions, time-dependent externally applied currents, and thermal gradients can all lead to a non-equilibrium state. A photon with an energy well above a Cooper pair can split into two high-energy quasiparticles. These decay by emitting phonons that can split more pairs into even more low-energy quasiparticles, creating a vast number of quasiparticles [15].

The Hubbard Kanamori (HK) model has emerged as one of the prototypes for transition metal oxide physics in the study of strongly correlated, many-electron systems [16]. The Hunds coupling terms in the multi-band model, which have a significant influence on high-temperature superconductivity, metal-insulator transitions, and other physical features, are multi-band in nature [17-18]. The populations of the quasi-electron and quasihole excitation branches, only one of the quasiparticle excitation modes that mesoscopic superconductivity deals with the charge mode is immediately accessible for conductance measurements. The experiment to examine the heat transfer in an open hybrid device built around an InAs nanowire that has been proximitized for superconductivity [19]. In a sense, research on quasiparticle excitations in superconductors dates back to the beginning of the phenomenon;

they make up the 'normal fluid' portion of the phenomenological two-fluid model, with the condensate of Cooper pairs serving as the superfluid component [20].

The research gap for the article lies in the unexplored relationship between Cooper pairs and the Josephson penetration depth. While the article discusses various aspects of superconductivity, including pair correlations, Josephson penetration depth, and the influence of external factors like magnetic fields and temperature, it does not specifically investigate or establish a connection between the pair correlation phenomena discussed and their impact on the Josephson penetration depth. Understanding how changes in pair correlations affect the Josephson penetration depth is crucial for gaining deeper insights into superconductivity and its practical applications. Further research in this area could help bridge this gap and contribute to our understanding of the fundamental properties of superconductors. This article is significant for its exploration of the relationship between pair correlations and the Josephson penetration depth in superconductors. This research addresses a crucial gap in our understanding of superconductivity, providing insights into the fundamental mechanisms at play. The findings have theoretical implications for the field and potential practical applications in designing superconducting devices with specific characteristics. Overall, this work contributes to advancing our knowledge of superconductivity and its practical utility.

METHOD

Pair-correlation function expressed in EQUATION (1) is taken from Tsuei and Kirtley, Das et al., and hence more detail can see in [21-22]. The pair correlation function related annihilation and creator operator with opposite momentum as cooper pair formation and quasiparticles environment as

$$\rho = \langle c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger} \rangle \langle c_{-k\downarrow} c_{k\uparrow} \rangle \quad (1)$$

Where $c$representannihilation and $c^{\dagger}$ represent creator operator of respective momentum, at T = 0 K,

$$\rho = \frac{\pi^2 \sin^2(2\theta_q)}{\epsilon^2 q^2 N_0^2 E_{ks}^2} \left\{1 + \frac{16\alpha^2 \pi k_F N_\sigma}{\epsilon q^2} \left[\left(v_- - sign(Rev_-)\sqrt{v_-^2 - 1} - v_+ + sign(Rev_+)\sqrt{v_+^2 - 1}\right)\right]\right\} \left\{1 + \frac{16\alpha^2 \pi k_F N_\sigma}{\epsilon q^2}\left(v_- - sign(Rev_-)\sqrt{v_-^2 - 1} - v_+ + sign(Rev_+)\sqrt{v_+^2 - 1}\right)\right\}^* \quad (2)$$

Where ϵ is dielectric constant, q is static factors, Here $v_{\pm} = \frac{\omega + i0}{qv_F} \pm \frac{q}{2k_F}$, $N_\sigma = \frac{m}{2\pi\hbar^2}$ is the 2D density of states of spin σ per unit area, $\boldsymbol{q}\perp \equiv (qy, -qx)$, ω *is* plasmon frequency and normalized plasmon frequency $\omega_0 = v_F k_F$, v_f is fermi velocity, $sign(Rev_+)$ is sign function with real value of $v_{\pm}$.

Case 1: For $v_- = v_+$ we have from EQUATION (2),

$$\rho_{(v_-=v_+)} = \frac{\pi^2 \sin^2(2\theta_q)}{\epsilon^2 q^2 N_0^2 E_{ks}^2} \tag{3}$$

At $T \neq 0K$,

$$\rho_T = \frac{\pi^2 \sin^2(2\theta_q)}{\epsilon^2 q^2 N_0^2 E_{ks}^2} \tanh^2\left(\frac{E_{ks}}{2k_B T}\right)\left\{1 + \frac{16\alpha^2 \pi k_F N_\sigma}{\epsilon q^2}\left(v_- - sign(Rev_-)\sqrt{v_-^2 - 1} - v_+ + sign(Rev_+)\sqrt{v_+^2 - 1}\right)\right\}\left\{1 + \frac{16\alpha^2 \pi k_F N_\sigma}{\epsilon q^2}\left(v_- - sign(Rev_-)\sqrt{v_-^2 - 1} - v_+ + sign(Rev_+)\sqrt{v_+^2 - 1}\right)\right\}^* \tag{4}$$

Case 2: For $v_- = v_+$ we have from EQUATION (4),

$$\rho_{T(v_-=v_+)} = \frac{\pi^2 \sin^2(2\theta_q)}{\epsilon^2 q^2 N_0^2 E_{ks}^2} \tanh^2\left(\frac{E_{ks}}{2k_B T}\right) \tag{5}$$

EQUATION (3) and (5) are the case 1 and case 2 of pair correlation with temperature independent and temperature dependent. The correlation shows in both case for constant number of quasiparticles is directly proportional to angle of cooper pair formation, inversely proportional to dielectric constant of material, q is static factor, $E_{ks}$is quasiparticle excitation energy for both cases but in case of 2 pair correlation is additional proportional to hyperbolic trigonometric function tan. On replacing T by critical temperature T_c of EQUATION (5) we get,

$$\rho_{T_c(v_-=v_+)} = \frac{\pi^2 \sin^2(2\theta_q)}{\epsilon^2 q^2 N_0^2 E_{ks}^2} \tanh^2\left(\frac{E_{ks}}{2k_B T_c}\right) \tag{6}$$

Also on solving EQUATION (6) we get

$$T_c = \frac{E_{ks}}{\tanh^{-1}\left(\frac{\sqrt{\epsilon^2 q^2 N_0^2 E_{ks}^2}\sqrt{\rho_{T_c(v_-=v_+)}}}{\sqrt{\pi^2 \sin^2(2\theta_q)}}\right) + i\pi n} \tag{7}$$

$$\tanh^{-1}\left(\frac{\sqrt{\epsilon^2 q^2 N_0^2 E_{ks}^2}\sqrt{\rho_{T_c(v_-=v_+)}}}{\sqrt{\pi^2 \sin^2(2\theta_q)}}\right) + i\pi n \neq 0, E_{ks} \neq 0, n\epsilon\mathcal{Z} \tag{8}$$

For n = 0,

$$T_c = \frac{E_{ks}}{\tanh^{-1}\left(\frac{\sqrt{\epsilon^2 q^2 N_0^2 E_{ks}^2}\sqrt{\rho_{T_c(v_-=v_+)}}}{\sqrt{\pi^2 \sin^2(2\theta_q)}}\right)} \tag{9}$$

Relation of Josephson penetration depth and London penetration depth

In recent years, Josephson junctions and superconducting quantum interference devices using graphene or other 2D materials as the weak link between superconductors have become a hot topic of research, both in terms of fundamental physics and potential applications. We derive a new expression for the Josephson penetration depth of such junctions and verify our assumptions by numerical simulations [23].

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0 J_C(L+2\lambda_L)}} \tag{10}$$

Where $\Phi_0 = \frac{\hbar}{2e}$ is the magnetic flux quantum, μ_0 is the free space permeability, L is the length of the junction (the thickness of the tunnel barrier), and λ_L is the London penetration depth. The equation is only valid if the London penetration depth is less than the superconducting electrode thickness, and it is not valid if the London penetration depth is comparable or greater than the superconducting electrode thickness. Using the relation of Josephson Penetration Depth with tunneling barrier and London Penetration Depth $\lambda_L(T) = \left(\frac{m}{ne^2\mu_0}\right)^{\frac{1}{2}}$, a new relation between JPD and LPD is extended as,

$$\lambda_J^2(\lambda_L) = \frac{\Phi_0}{2\pi\mu_0 I_C\left[L+2\left(\frac{m}{ne^2\mu_0}\right)^{\frac{1}{2}}\right]} \tag{11}$$

Here, $\lambda_L(T) = \left(\frac{m}{ne^2\mu_0}\right)^{\frac{1}{2}}$ is London penetration depth. Also, $\xi(T) = \frac{\hbar v f}{\pi \nabla}$ is Coherence length and $H_C(T) = \frac{\Phi_0}{2\sqrt{2}\lambda_L\xi}$ is the magnetic field at temperature T. On substituting the value of magnetic field relation, coherence length, and London penetration depth. The other form of EQUATION (9) is obtained, with the new relation as,

$$\lambda_J^2(\lambda_L, H_C) = \frac{\Phi_0}{2\pi\mu_0 I_C\left(L+2\,\frac{\Phi_0}{2\sqrt{2}H_c(T)\frac{\hbar v_f}{\pi\nabla}}\right)} \tag{12}$$

$$\lambda_J^2(\lambda_L, H_C) = \frac{\Phi_0}{2\pi\mu_0 I_C\left(L+\frac{\pi\Phi_0\nabla}{\sqrt{2}H_c(T)\hbar v_f}\right)} \tag{13}$$

This relation shows how the fermi velocity effect, energy gap, and Magnetic field related to Josephson penetration depth. Using relation of critical magnetic field and transition temperature of superconductor EQUATION (13) become,

$$\lambda_J^2(\lambda_L, H_C) = \frac{\Phi_0}{2\pi\mu_0 I_C\left(L + \frac{\pi\Phi_0\nabla}{\sqrt{2}\left(1-\frac{T^2}{T_C^2}\right)H_c(0)\hbar v_f}\right)} \tag{14}$$

Now using value of $T_c = \frac{E_{ks}}{\tanh^{-1}\left(\frac{\sqrt{\epsilon^2 q^2 N_0^2 E_{ks}^2}\sqrt{\rho_{T_C(v_-=v_+)}}}{\sqrt{\pi^2 \sin^2(2\theta_q)}}\right)}$ in EQUATION (14) and taking the ratio of Josephson Penetration depth to normal Josephson Penetration depth (λ_J) and Josephson Penetration depth to electron-phonon and transition temperature of superconductor $\lambda_J^2(\lambda_L, H_C)$, as,

$$\frac{\lambda_J^2(\lambda_L, H_C)}{\lambda_J^2} = \frac{\left[\frac{\Phi_0}{2\pi\mu_0 I_C\left(L + \frac{\pi\Phi_0\nabla}{\sqrt{2}\left(1-\frac{T^2}{T_C^2}\right)H_c(0)\hbar v_f}\right)}\right]}{\frac{\Phi_0}{2\pi\mu_0 t I_c}} \tag{15}$$

$$\lambda_J^2(\lambda_L, H_C) = \lambda^2 \lambda_J^2 \tag{16}$$

Where $\lambda_J^2(\lambda_L, H_C) = \frac{t}{\left(L + \frac{\pi\Phi_0\nabla}{\sqrt{2}\left(1-\frac{T^2}{T_C^2}\right)H_C(0)\hbar v_f}\right)}$.

New Sine-Gordon Equation with Mixed JPD and LPD Unperturbedsine-Gordon Equation

The unperturbed Sine-Gordon equation for superconductor is given by,

$$\frac{\partial^2\varphi}{\partial t^2} - \omega_p^2 \lambda_{LJ}^2 \frac{\partial^2\varphi}{\partial x^2} + \omega_p^2 \sin\varphi = 0 \tag{17}$$

Where λ_{LJ}^2 obtained in EQUATION (16) and hence EQUATION (17) is interlinked with LPD. This shows that sine-Gordon equation is depend and interlink JPD and phase of sine-Gordon equation. Let us consider $\omega_p^2 \lambda_{LJ}^2 = k^2$, a=$\omega_p^2$, $\omega_p^2 = \frac{2\pi I_c}{\Phi_0 C} => \omega_p = \sqrt{\frac{2\pi I_c}{\Phi_0 C}}$

$$\frac{\partial^2\varphi}{\partial t^2} - k^2 \frac{\partial^2\varphi}{\partial x^2} + a\sin\varphi = 0 \tag{18}$$

or,

$$\varphi_{tt} - k^2 \varphi_{xx} + a\sin\varphi = 0 \tag{19}$$

Consider a long Josephson junction, which is made up of two relatively long strips of superconducting material separated by a very thin dielectric of a thickness (d). This device's element of length dx is electrically equivalent to a circuit with capacitance per unit length

$C = K\varepsilon_0 a/d$, where K is the dielectric constant and$a$ is the width of the superconductor strip [24]. Analytical progress toward understanding the dynamics of kinks can be made using a known exact solution, $\Phi(x)$, of the unperturbed equation that determines the characteristic shape of the structure. An approximate formula or ansatz for the configuration at time t is $\varphi(x,t) = \Phi(x - X(t))$, where $X(t)$, is the position of the center of the structure at time t, is regarded as a dynamical variable or collective coordinate [25]. The kink Ansatz that will be used to solve this EQUATION (19),

$$\varphi(x,t) = 4\arctan\{A\exp[B(x - vt)]\} \tag{20}$$

The variable E will be used henceforth, where$E = A\exp[B(x - vt)]$. Inserting EQUATION (20) into EQUATION (19) yields,

$$4\frac{E - E^3}{(1 + E^2)^2}\left[B^2\left(v^2 - \omega_p^2\lambda_{LJ}^2\right) + \omega_p^2\right] = 0 \tag{21}$$

Solving for B gives,

$$B = \pm\sqrt{\frac{\omega_p^2}{\omega_p^2\lambda_{LJ}^2 - v^2}} \tag{22}$$

It turns out that $A = \pm e^{-Bx_0}$ represents the starting location of the soliton at x_0. Positive A represents a bright soliton, whereas negative A represents a dark soliton. The sign of $\mathbf{A.B}$ determines the direction of the internal twist in the kink. These things hold true whenever the solution structure is an arctangent of an exponential. The variable R will be used to hold all of the perturbation terms, where,

$$R = \beta q_t + \gamma q_x + \delta q_{xt} + \lambda_c q_{tt} + \sigma q_{xxt} + \nu_d q_{xxxx} \tag{23}$$

In Josephson junctions, β represents the dissipative losses of electrons tunneling across a dielectric barrier, γ comes about from an inhomogeneous part of the local inductance, δ accounts for the diffusion, λ_c results from an inhomogeneity of the capacitance, σ arises due to current losses along the barrier, and ν_d contains the higher-order spatial dispersion.

Perturbed Sine-Gordon Equation

The perturbed Sine-Gordon equation for superconductor is given by,

$$\frac{\partial^2\varphi}{\partial t^2} - \omega_p^2\lambda_{LJ}^2\frac{\partial^2\varphi}{\partial x^2} + \omega_p^2\sin\varphi = R \tag{24}$$

The solution of the perturbed Sine-Gordon equation is,

$$\varphi(x,t) = 4\arctan\left\{\exp\left[\pm\sqrt{\frac{\omega_p^2}{\omega_p^2\lambda_{LJ}^2 - \delta v_d - (1 - \lambda_c)v_d^2}}(x - x_0 - v_d t)\right]\right\} \tag{25}$$

Recent research has focused on Josephson junctions and superconducting quantum interference devices (SQUIDs) utilizing graphene or other 2D materials as weak links, with applications in both fundamental physics and potential technologies [26]. Ultrawide Josephson junctions based on chemical-vapor-deposition graphene have shown uniform critical current distribution. A novel superconducting memory cell, encoding state via Josephson vortices, has been developed, promising energy-efficient and non-destructive control. Integration into coplanar resonators enables scalability and compatibility with superconducting microwave technologies. Additionally, NbN/Au nanogaps bridged with Al superconductor exhibit high critical current density and Josephson coupling, highlighting the importance of superconducting barriers for achieving high Josephson current in nanodevices, crucial for superconducting circuits with high integration density [27-28].

RESULT AND DISCUSSION

The developed equation in methodology section EQUATION (14) and EQUATION (9) and to study the nature of developed equation was computed using MATLAB and for this data form [26-28] and study the nature of JPD with temperature and with fermi velocity as shown in FIGURE 1 and 2 respectively. The nature of Josephson penetration Depth (JPD) to electron-phonon and transition temperature in FIGURE 1 shoes that JPD decrease with increasing temperature. This is because the temperature causes the expansion of superconductors and hence the JPD decreases. In high temperature, the cooper pairs go break down and hence JPD decreases sharply. In general, the JPD is exponentially decreases with temperature. This phenomenon arises due to the impact of temperature on the lattice structure of superconducting materials. At higher temperatures, thermal energy is imparted to the lattice, causing it to vibrate more vigorously. This increased vibrational energy leads to a greater average separation between the lattice atoms, resulting in an expansion of the superconducting material. As the lattice expands, the characteristic penetration depth of the superconducting state decreases. This is because the coherence of the superconducting electron pairs is diminished, making it harder for them to maintain their phase relationship over longer distances. Consequently, the JPD, which quantifies the depth at which the superconducting phase is perturbed by an external magnetic field, decreases with increasing temperature.

At high temperatures, the thermal energy exceeds the binding energy of the Cooper pairs. As a result, some of these pairs are disrupted, leading to a reduction in their coherence. The Cooper pairs are the cornerstone of superconductivity, as they are responsible for the zero-resistance and expulsion of magnetic fields in superconductors. When the Cooper pairs break down, the superconducting state is compromised, and the characteristic properties associated with superconductivity, including the JPD, undergo a sharp decline. This is because the loss of coherence among the electron pairs hinders the ability of the superconductor to maintain a robust superconducting state. The exponential decrease in JPD with temperature can be attributed to the multiplicative nature of the physical processes involved. The expansion of the lattice due to thermal effects and the breakdown of Cooper pairs are interdependent phenomena that act together to influence the superconducting state. The exponential trend highlights that as temperature continues to rise, the combined effects of lattice expansion and Cooper pair breakdown have a compounded impact on the JPD. This underscores the sensitivity of superconducting properties to temperature variations and provides insights into how the material behaves across a range of temperatures.

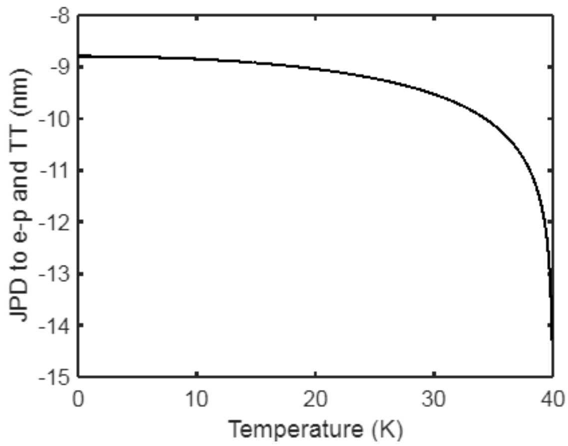

FIGURE 1. JPD to e-p and TT with transition temperature.

The JPD increase with increasing Fermi velocity as shown in FIGURE 2. The JPD is almost constant when electron speed approaches to speed of light. The JPD increase sharply when speed of Fermi elections is lower. In general, the nature of JPD is exponentially increasing with Fermi velocity. The Fermi velocity represents the velocity at which electrons near the Fermi level move within a material. When the Fermi velocity is higher, electrons are more mobile, allowing them to respond more quickly to changes in the electromagnetic environment. This enhanced mobility enables them to maintain their superconducting coherence over a larger distance, which in turn increases the Josephson Penetration Depth (JPD).As the Fermi velocity increases, electrons can effectively “keep up” with the superconducting state's phase relationship over a greater distance. This leads to an expansion of the region over which the superconducting state remains robust, resulting in an increased JPD.

As electrons approach relativistic speeds (close to the speed of light), they start to exhibit relativistic effects. At such high velocities, the relativistic mass increase and time dilation come into play. These relativistic effects can alter the behavior of electrons in a superconducting material. Specifically, the increased mass and time dilation can influence the coherence of Cooper pairs. When electron speeds approach the speed of light, relativistic effects become more dominant, potentially counteracting the increased mobility associated with higher Fermi velocities. This can lead to a plateau in the increase of JPD, resulting in a near-constant value. At lower speeds of Fermi electrons, the mobility of the electrons is restricted. This restriction hinders their ability to maintain coherence over long distances, reducing the superconducting state's characteristic penetration depth. As a result, the JPD increases sharply. When electron speeds are lower, they struggle to keep up with the dynamic changes in the superconducting state, leading to a more confined region where superconductivity is maintained. This sharp increase in JPD indicates the heightened sensitivity of the superconducting state to electron mobility.

The exponential increase in JPD with Fermi velocity underscores the multiplicative nature of the physical processes involved. As Fermi velocity increases, the enhanced mobility of electrons amplifies their ability to maintain coherence, resulting in a compounded effect on the JPD. The exponential trend demonstrates that small changes in Fermi velocity can lead to significant alterations in the superconducting state's characteristic properties. This highlights the critical role of electron mobility in influencing the behavior of superconducting materials.

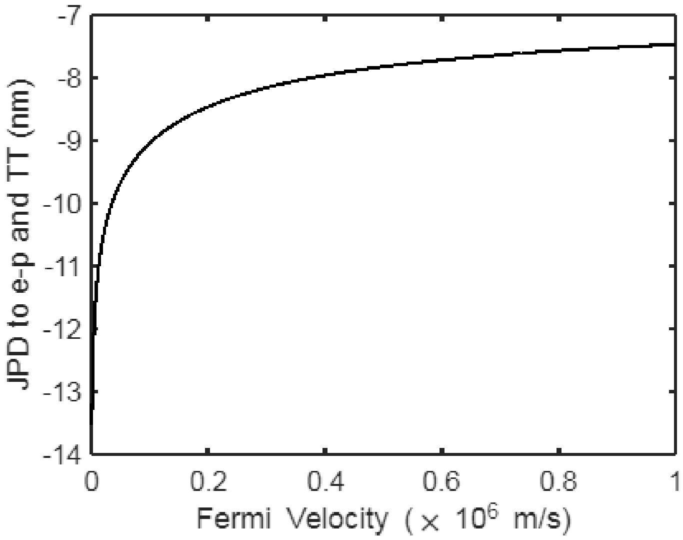

FIGURE 2. JPD to e-p and TT with Fermi velocity.

Critical Temperature

The nature of critical temperature of superconductors are shown in FIGURE 3. The nature shows at zero radian phase angle the temperature is zero while around ±0.7 radian the critical temperature is high. At zero radian phase angle, the critical temperature is observed to be zero. This phenomenon is indicative of a specific property in certain superconducting materials, known as phase transition. In these materials, the transition to a superconducting state occurs abruptly at a critical temperature, where the resistance drops to zero. When the phase angle is zero, the superconducting material is in a specific state where the quantum mechanical properties of the material are aligned in such a way that the resistance drops to zero at absolute zero temperature. This is a characteristic feature of certain types of superconductors, known as type I superconductors. Around ±0.7 radian phase angle, the critical temperature is observed to be high. This can be attributed to the specific characteristics of the material and its crystalline structure. In some superconductors, the critical temperature is elevated due to factors such as the density of states near the Fermi level and the strength of the electron-phonon interactions. The high critical temperature at around ±0.7 radian indicates that the superconducting state in these materials can be achieved at temperatures significantly above absolute zero. This is particularly significant for practical applications where superconducting behavior is desired at higher temperatures.

The observed variations in critical temperature at different phase angles highlight the diversity in superconducting behavior across different materials and conditions. Understanding the relationship between phase angle and critical temperature is crucial for tailoring superconductors to specific applications, as it allows for the selection of materials with the most suitable superconducting properties for a given scenario. This finding contributes valuable insights to the field of superconductivity and enables the design of materials with enhanced superconducting capabilities.

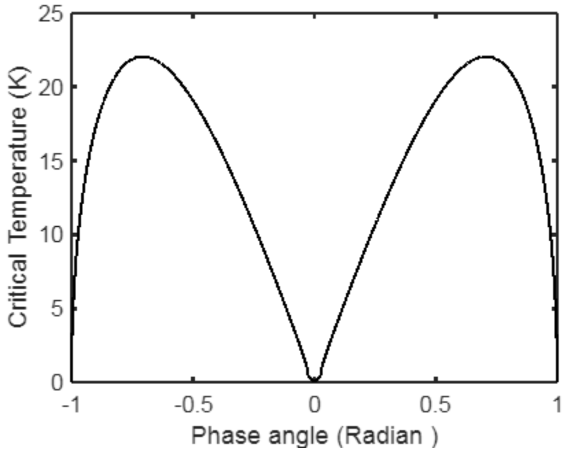

FIGURE 3. Nature of critical temperature with Phase angle.

FIGURE 4 shows that critical temperature decrease with increasing in cooper pair correlation. With lower number of pair correlation region the decreasing critical temperature is sharply while behind it is slow. In general the nature of critical temperature is like exponentially decay. Cooper pairs play a pivotal role in superconductivity. They are formed by the interaction between electrons and lattice vibrations (phonons) in the material. As the cooper pair correlation increases, it implies that there is a higher degree of coherence among these pairs. This coherence is crucial for the maintenance of the superconducting state. When the cooper pair correlation decreases, it signifies a reduction in this coherence, making it more challenging for the material to sustain superconductivity. With a higher level of cooper pair correlation, the material can effectively maintain its superconducting state at higher temperatures, resulting in an elevated critical temperature. Conversely, a decrease in cooper pair correlation indicates a reduced ability to maintain the superconducting state, leading to a lower critical temperature. When the cooper pair correlation is low, there is less coherence among the electron pairs. This means that the material is less capable of maintaining the superconducting state, even at lower temperatures. Consequently, a small decrease in pair correlation can have a significant impact on the critical temperature. In regions with lower cooper pair correlation, the material's ability to sustain the superconducting state is highly sensitive to changes in the correlation level. This results in a sharp decrease in critical temperature when the cooper pair correlation diminishes. The exponential decay nature of the critical temperature with respect to cooper pair correlation arises from the multiplicative effect of the physical processes involved. As the cooper pair correlation decreases, the coherence and stability of the superconducting state are progressively compromised. This leads to an exponential decline in the critical temperature. The exponential decay trend underscores the profound influence of cooper pair correlation on the superconducting state. It indicates that even small changes in pair correlation can have a substantial impact on the material's ability to exhibit superconductivity. In summary, the detailed analysis reveals how the cooper pair correlation profoundly influences the critical temperature of superconductors.

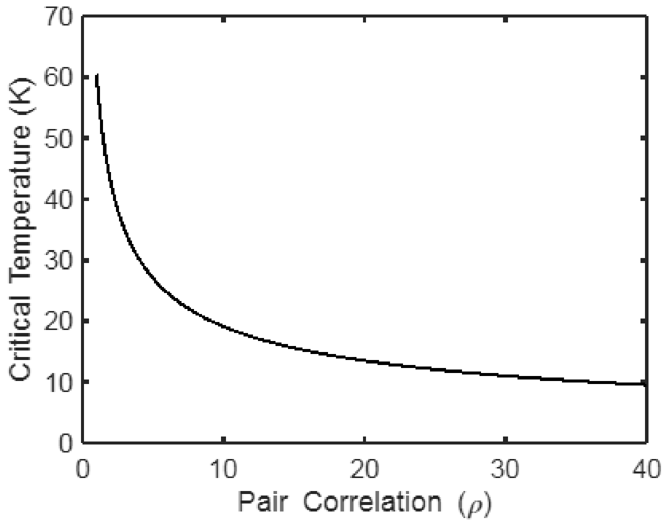

FIGURE 4. Nature of critical temperature with pair correlation.

CONCLUSION

In conclusion, this study sheds light on the intricate relationship between various parameters and the critical temperature of superconductors. The findings reveal that critical temperature decreases as cooper pair correlation increases, signifying the pivotal role of coherence among electron pairs in sustaining the superconducting state. Moreover, regions with lower pair correlation exhibit a sharp decline in critical temperature, highlighting their heightened sensitivity to changes in correlation levels. The observed exponential decay nature underscores the profound impact of cooper pair correlation on superconductivity, emphasizing its significance in material design and optimization. These insights provide a valuable roadmap for tailoring superconducting materials to specific applications like SQUIDs and MRI, enabling the development of materials with enhanced superconducting properties. This study not only advances our fundamental understanding of superconductivity but also offers practical implications for the design and engineering of materials for a wide range of technological applications.

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Conflict of Interest: No conflict of Interest.

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