JURNAL ILMIAH WAHANA AKUNTANSI
Vol 15 (1) 2020, 46-56
http://journal.unj/unj/index.php/wahana-akuntansi

MODELING THE VOLATILITY FOR LONG TERM INTEREST RATE RETURNS IN THE NIGERIA BOND MARKET USING CONDITIONALY HETEROSCEDASTIC MODELS

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ARTICLE INFO

Keywords:
Volatility Models, Maximum likelihood, Long term interest rate, GED, Bond market

ABSTRACT

Investigating the volatility of financial assets is fundamental to risk management. This study used generalized Autoregressive Con-ditional Heteroscedastic Volatility models to evaluate the volatility of the long term inter-est rate of Nigeria's financial market. We al-so incorporated three innovations distribu-tions viz: the Gaussian, the student-t, and the Generalized Error Distribution (GED) in the modeling process under the maximum likeli-hood estimation method. The results show that GARCH (GED) is the most performing model for describing the volatility of three and twenty-year interest rate returns while TARCH (GED) is the most suitable model for describing the volatility of five and ten-year interest rate returns in Nigeria. The pre-ferred models will help in the development of tools for effective risk management by moni-toring the behavior of long term interest rates.

How to Cite:
Dallah Hamadu and Olaniyan Sunday Michael. (2020). Modeling the Volatility for Long Term Interest Rate Re-turns in the Nigeria Bond Market using Conditionaly Het-eroscedastic Models. Jurnal Ilmiah Wahana Akuntansi, 15(1), 46-56 https://doi.org/10.21009/wahana.15.014

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ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

INTRODUCTION

Volatility in the financial market has been the attention of business and policy-makers due to its importance in policymaking, risk management, securities analysis, and pricing (Dayioğlu, 2012). Accurate volatility modeling is fundamental to good risk management while better risk management practices lead to better stability of the economy with evident social benefits.

Modeling the volatility of the bond markets (Long term interest rate) is central in risk management because the long term interest rate constitutes the larger part of happenings in the non-financial economy and financial markets i.e. monetary policy and the financial aspects of fis-cal policy (Friedman, 1980). Term structure of interest rate volatility especially the long term has gotten significant consideration from both scholars and professionals in recent years. It is essential to capture the volatility of term interest rates because they affect businesses, borrowing costs and investment account earnings. The most successful and popular volatility models are the GARCH (Generalized Autoregressive Condition-al Heteroscedastic) model which was proposed by Bollerslev (1986) who generalized the ARCH (Autoregressive Conditional Heteroscedastic) models by Engle (1982). Multiple extensions of the standard ARCH function have been proposed to capture additional stylized facts observed in financial markets (Ardiaa, Bluteaua, Boudt & Catania, 2017). The GARCH models recognize that there may be important nonlinearities, asymmetries, and long memory properties in the volatility process (Ardiaa, Bluteaua, Boudt & Catania, 2017). GARCH models also take into account the time-varying volatility phenomenon over a long period which is the most commonly used model in the family of GARCH models and has indeed proven to be very useful in describing a wide vari-ety of financial market data (Sarkar & Mukho-padhyay, 2005).

To cover specific volatility features like the well-known leverage effect and other asymmetries in financial returns (Black, 1976; Christie, 1982 cited by Reher & Wilfling, 2011, Petrică & Stancu, 2017), Nelson (1991) extended the usual GARCH model known as the exponential GARCH (EGARCH) to capture the leverage effect alongside volatility. Other extensions of GARCH have been suggested to capture asymmetric responses in the conditional variance to positive and negative shocks. Glosten et al. (1993) and Zakoian (1994) have proposed utilizing the threshold GARCH (TGARCH), model. The standard deviation GARCH was proposed by Taylor (1986) and Schwert (1989) so as to model the standard deviation rather than the variance. This model, alongside a few different models, is generalized with the power ARCH specification in (Ding Granger, & Engle, 1993). In the power ARCH (PARCH) model, the power parameter of the standard deviation can be evaluated instead, and the optional parameters are added to capture asymmetry (Mukhopadhyay & Sarkar, 2013).

To the best of our knowledge, little or no known study has considered the issues of "leverage effect" and excess kurtosis on long term interest rate data in Nigeria. To this effect, this study used ARCH, GARCH, TARCH, EGARCH, and PARCH models to model the volatility of long term interest rates in Nigeria and to compare their performance. The study also incorporated three innovations distributions such as the Gaussian, the student-t, and the Generalized Error Distribution (GED) to the volatility models.

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

LITERATURE REVIEW

This section gives an overview of the Nigerian bond market and empirical review of related literature of the study

Overview of the Nigerian Bond Market

Bonds are the basic type of tradable financial contract by which corporations and governments tap into the capital available from investors (Grasselli & Hurd, 2015). The issuer of a bond presents the bond as a guarantee to make accessible regular income installments to the investor. These income payments are coupons that pay coupons two times every year (semi-yearly coupon bonds) and (yearly coupon bonds). Bonds that make no coupon payments are known as zero-coupon bonds (NSE, 2020). The Nigerian bond market is classified as the second most liquefied market in sub-Saharan Africa (Ajayi, 2013). Nigerian bond market is regarded by many Africa market as an ideal to learn from to improve their domestic bond markets (Lartey & Li, 2018a).

FGN Bonds are debt securities of the Federal Government of Nigeria (FGN) issued by the Debt Management Office (DMO) for and on behalf of the Federal Government. Before the foundation of the Debt Management Office (DMO) in 2000, Nigeria's public debt was overseen by various government offices in a clumsy way. This dispersion made issues that achieved a genuine strain on the nation's debt portfolio and economy development. The foundation of the DMO denoted the initiation of the systematization and professionalization of public debt management in Nigeria (DMO, 2020). Purchasing FGN securities suggests loaning to the FGN for a predefined period and are considered as the most secure of all investments in domestic debt market since it is sponsored by the 'full trust and credit' of the Federal Government, and as such it is delegated a risk -free debt instrument, implying that it is sure that interest and principal will be paid as and when due. The summed up highlights of FGN Bonds incorporates as revealed by the Debt Management Office of Nigeria (2020) incorporates:

i. Denomination: least subscription of *N*10,000 plus several of *N*1,000 subsequently.

ii. Interest payment: Most FGN bonds have fixed interest rates payable semi-every year. Some FGN bonds (for example third and fourth tranches of the first FGN securities) have floating rate of interest which change around a reference rate (NTB rates) in light of indicated parameters. There are likewise zero-coupon bonds (not yet in issue in Nigeria) whereby both interest and principal are repaid at the final maturity date of the bond.

iii. Tenor: Minimum of two (2) years. There are bonds with maturities of 3, 5, 7, and 10 years in issue and may have bonds with maturities of 15, 20, 30 years or more and for the future

iv. Default Risk: FGN bonds as an obligation are the most secure venture instrument since they have no default.

Empirical Review of Related Literature

Several works have been done on modeling the volatility of term structure of interest rate especially the short term interest rate. For instance, Li, Tahir, Ain and Yousaf (2020) analyzed the volatility of the short term interest rate of the Pakistani financial market utilizing GARCH and E-GARCH models on a monthly data of T-bills covering the period January 2005 to December 2012. The outcome shows that the GARCH model is the most appropriate model to predict the volatility

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

behavior of short term interest rates when con-trasted with the E-GARCH model. Olweny (2011) modeled the volatility of short-term inter-est rates in Kenya using the monthly averages of the 91-day T-BILL rate data which were gotten from the Central Bank of Kenya between August 1991 and December 2007. The result revealed that the GARCH model is a suitable candidate for exploring the volatility of short rates in Ken-ya, rather than ARCH models. Hou and Suardi (2011) utilized a semi-parametric technique to assess the diffusion process of short-term interest rates. The Monte Carlo study shows that the semi -parametric methodology generates more precise volatility estimates than the models that accom-modate asymmetry, level effect, and serial de-pendence in the conditional variance. Turan (2000) tested the performance of stochastic vola-tility models of the short-term interest rate by de-veloping a nonlinear asymmetric framework that takes into consideration for comparisons of non-nested models featuring conditional heteroske-dasticity and sensitivity of the volatility process to interest rate levels. Two-factor stochastic vola-tility models are tested against the famous contin-uous-time and symmetric and asymmetric GARCH models. The newly proposed model out-performs the existing as a result of the asymmet-ric drift of the short rate, and the presence of non-linearity, asymmetry, GARCH, and level effects in its volatility. Charlotte (2005) uses a multivari-ate level-GARCH model for the *long-rate* and the term-structure *spread.* The findings show that long-rate variance exhibits heteroskedasticity ef-fects and level effects following the square-root model. The spread variance exhibits heteroske-dasticity effects but no level effects. The level-GARCH model is preferred above the GARCH model and the level model. Literature has shown pieces of proof that asset returns display volatility clustering, leptokurtosis, and asymmetry. However, few studies have investigated the volatility of bond yields (long term or short term interest rate) in Nigeria. Most of works in Nigeria are centered on stock volatility and exchange rates volatility. For example, Bichi, Dikko and Nagwai (2016) employed the two most popularly use Multivariate GARCH models – the Baba-Engle-Kraft-Kroner (BEKK) and the Dynamic Conditional Correla-tion (DCC) model in modeling the volatility spill-over between the Nigerian Stock and Bond Mar-ket. The study revealed that the own past shocks affect the current volatility of the Nigeria stock market and a bidirectional volatility spillover be-tween Nigerian stock and bond markets. The DCC is the most suitable model for modeling intra-national volatility transmission for the Nigerian stock and bond markets. Dallah and Ibiwoye (2010) who modeled and forecasted the volatility of the Nigerian insurance stocks returns shows that EGARCH (1, 1) was the most suitable in modeling stock returns as it outclasses other vola-tility models in terms of model performance crite-ria. The work of Olowe (2009) revealed stock market crash of 2008 was found to have impacted to high volatility persistence in the Nigerian stock market particularly during the global financial cri-sis. Bala and Asemota (2013) examined exchange -rate volatility with GARCH models using month-ly exchange-rate returns series from 1985-2011 for Naira/US dollar and 2004-2011 for Naira/US dollar and 2004-2011 for Naira/British Pounds

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

and Naira/Euro returns. The findings revealed the presence of volatility in the selected currencies and also most of the asymmetric models rejected the existence of leverage excluding for models with volatility break. Emenike (2010) modeled GARCH (1, 1) and GJR-GARCH (1, 1) to the All -share-index in Nigeria and the findings indicated that the NSE returns is described by leverage effects and volatility persistence. The findings of Babatunde (2013) in Nigeria's stock market volatility and economic growth using the EGARCH model' revealed that the volatility shock is quite untiring and this might alter the growth of the Nigerian economy. Asemota and Ekejiuba (2017) made use of GARCH models to investigate the volatility of the six bank's equity returns in Nigeria. Findings showed the existence of ARCH effect in some bank's equity returns. Besides, the estimated models could not find evidence of leverage effect.

MATERIALS AND METHOS Data for the study

The data used for this research work were obtained from Meristem Securities Limited. They are a historical set of 856 interest rate data from Nigeria Government Securities. The sample period extends from 5th January 2015 to 23rd February 2018 considering the long-term interest rates (Nigeria government bond yields) for four different maturities of 3-year (3YR), 5-year (5YR), 10-year (10YR) and 20-year (20YR).

Analysis techniques

The techniques adopted includes the calculation of log returns, maximum likelihood estimates of GARCH models with different conditional distributions assumptions and also the model performance evaluation criteria. The analysis techniques are presented below:

Returns

The first step is to obtain the daily long-term interest rates and compute the compound returns simply by using the natural logarithm of long-term interest rates of the nth day over (n-1)th day. This can be express mathematically as:

$$Returns = \log_e\left(\frac{I_{t+1}}{I_t}\right) \quad (1)$$

Distribution

Because financial time series are generally fat-tailed, the use of normal distributions might be limited. As a result of this, the student-t and GED distributions are also used. Hence, we have the following results for the log-likelihood function applied to a sample of T observations

For normally distributed standardized innovations:

$$L_{Normal} = -\frac{1}{2}\sum_{t=1}^{T}\left(\ln(2\pi) + \ln(\delta_t^2) + z_t\right) \quad (2)$$

For standardized t-distributed innovations:

$$L_{Student-t} = \ln\left(\Gamma\left(\frac{v+1}{2}\right)\right) - \ln\left(\Gamma\left(\frac{v}{2}\right)\right) - \frac{1}{2}\ln(\pi(v-2)) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln(\delta_t^2) + (1+v)\ln\left(1+\left(\frac{z_t^2}{v-2}\right)\right)\right) \quad (3)$$

$$L_{GED} = \sum_{t=1}^{T}\left(\ln\left(\frac{v}{\lambda_v}\right)\right) - \frac{1}{2}\left|\frac{z_t}{\lambda_v}\right|^{v} - (1+v^{-1})\ln(2) - \ln\left(\Gamma\left(\frac{1}{v}\right)\right) - \frac{1}{2}\ln(\delta_t^2) \quad (4)$$

Where T number of data, v degree of freedom,

$2 < v \leq \infty, \Gamma(\ )$ gamma function,

$$\lambda_v = \sqrt{\frac{\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)} 2^{\frac{-2}{v}}}$$,

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

Conditionally Heteroscedastic model *ARCH*

The general form of the ARCH (q) model is as follows:

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$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (5)$$

where:

σ_t^2 – the conditional variance of the innovations (errors) at time t;

ω – the constant term;

ε_{t-i}^2 – the squared error at time t-i;

α_i – ARCH terms i.e. volatility shocks from pre-vious periods.

GARCH

The general form of the GARCH (p,q) model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (6)$$

where:

ω – the constant term;

α_i – ARCH terms i.e. volatility shocks from pri-or periods.

β_j – GARCH terms i.e. the persistence of vola-tility;

p – the number of lagged conditional variance terms (δ_t^2);

q– the number of lagged errors (ε_t^2).

EGARCH

The EGARCH (p,q) model is given by:

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{q} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\delta_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\delta_{t-i}} \right) + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2) \quad (7)$$

where γ_i represents the asymmetry parameter (leverage effect).

TARCH

The TARCH (p,q) model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma_i \varepsilon_{t-1}^2 I_{t-1} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (8)$$

where *I* - represents the indicator function.

PARCH

The conditional variance of Power ARCH or PARCH (p,d,q) is given as

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-1}| + \gamma_i \varepsilon_{t-1})^{\delta} + \sum_{j=1}^{p} \beta_j (\delta_{t-j}^{\delta}) \quad (9)$$

α_i – the standard ARCH term;

β_j – the standard GARCH term;

γ_i – the leverage parameter ($|\gamma_i| < 0$);

δ – the parameter for the power term (δ >0).

Model Performance Evaluation Criteria

The model evaluation technique was based standard criteria including on Log-likelihood (-2*LL*), Akaike Information Criteria (= −2 + 2), Bayesian Information Criteria (= −2 + (ln)) and the Hannan-Quinn Criteria (*HQC* = −2 + 2 ln(ln)), where symbolizes the no of parameters used in the regression model, represent the sample volume while is the log-

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

likelihood function. The lower the value of AIC, BIC, and HQC, the better the performance of the model. While the higher the LL the better the performance of the model.

RESULT AND DISCUSSION

This section presents the descriptive statistics of data, the maximum likelihood estimates of GARCH models with different conditional distributions and also the model performance of the Heteroscedastic model. Figure 1 and 2 depicts the long-term interest rates data and long-term interest rates return respectively.

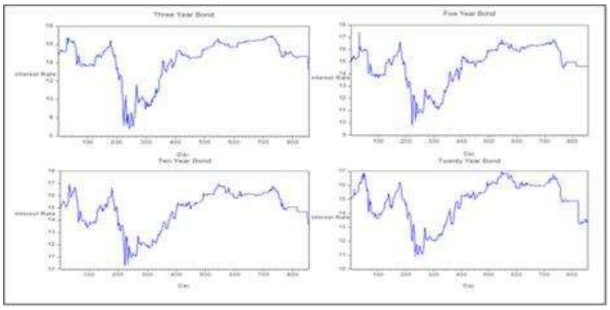

Figure 1: Movement of Long Term Interest rate Data

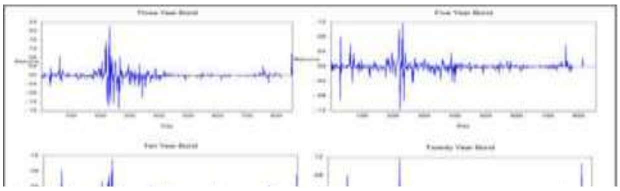

Figure 2: Movement of Long Term Interest rate Returns

Table 1
Descriptive Statistics of Long-Term Interest Rate Returns

Table 1 summarizes the data and describes the sample characteristics of the long term interest rate. The Table shows that the mean returns of the 3YR, 5YR, 10YR, and 20YR maturities clusters around 0.0001366, 0.0000528, 0.0001118, and 0.0001373 respectively. The implication of this is that all the returns display a high level of consistency as their average values are contained by the maximum and the minimum values of these returns. Results in Table 1 demonstrated that the returns for all the maturities are positively skewed and Kurtosis coefficients exhibited a leptokurtic distribution (Kurtosis>3), inferring a fat-tailed empirical distribution of the returns over the periods. The kurtosis result depicted that a fat tailed distribution such as the student-t or a Generalized Error Distribution (GED) would make improved results than just a normal distribution (Dayioğlu, 2012). All the minimum returns are negative while the maximum are positive as evidenced in Figure 2.

Table 2
Maximum Likelihood Estimates of GARCH Models with different Conditional Distributions

Maturities	Models	Parameters	Normal		t		GED	
			Estimate	Prob	Estimate	Prob	Estimate	Prob
Three Years	ARCH	ω	0.000198	0.0000	0.009629	0.0307	0.000269	0.0432
		α	1.520401	0.0000	1671.094	0.0342	0.777561	0.0675
	GARCH	ω	7.32E-06	0.0000	1.17E-13	0.5115	1.91E-05	0.5082
		α	0.167831	0.0000	0.974307	0.0000	0.565476	0.4019
		β	0.830383	0.0000	0.549986	0.0000	0.997638	0.0000
	TARCH	ω	6.83E-06	0.0000	5.63E-14	0.5131	1.50E-05	0.0597
		α	0.22103	0.0000	0.670504	0.0000	0.609721	0.1065
		γ	-0.13005	0.0000	0.444467	0.0110	0.864275	0.2140
		β	0.841694	0.0000	0.537236	0.0000	0.688737	0.0000
	EGARCH	ω	-0.08144	0.0000	-0.16028	0.0000	-7.48649	0.0000
		α	0.028594	0.0000	0.624581	0.0536	0.544724	0.0019
		γ	0.102663	0.0000	-0.20125	0.0962	-0.0064	0.9685
		β	0.991647	0.0000	0.993227	0.0000	-0.01231	0.9532
	PARCH	ω	1.27E-09	0.6113	-3.96E-09	0.9241	0.000311	0.5472
		α	0.111732	0.0000	0.648021	0.0000	0.469203	0.2412
		γ	-0.16604	0.0000	-0.02343	0.6558	0.329376	0.2610
		β	0.764034	0.0000	0.707688	0.0000	0.969274	0.0000
		δ	3.798854	0.0000	1.046193	0.0000	1.287691	0.0079
Five Years	ARCH	ω	4.63E-05	0.0000	0.005474	0.3534	0.000165	0.0000
		α	2.895521	0.0000	2013.007	0.3580	0.560075	0.0253
	GARCH	ω	0.000108	0.0000	7.97E-15	0.7415	1.31E-05	0.0082
		α	0.15	0.0000	0.674281	0.0000	0.614665	0.0017
		β	0.6	0.0000	0.50999	0.0000	0.730149	0.0000
	TARCH	ω	1.02E-05	0.0000	-9.19E-09	0.0000	0.000108	0.5581
		α	0.430643	0.0000	0.15915	0.0000	0.151151	0.8871
		γ	0.075701	0.1882	-0.08278	0.0004	-0.11694	0.9482
		β	0.547766	0.0000	0.833694	0.0000	1.014763	0.0000
	EGARCH	ω	-1.38095	0.0000	-0.26163	0.0000	-8.43288	0.3159
		α	0.544753	0.0000	3.03725	0.0000	0.115805	0.3679
		γ	-0.02325	0.1088	-3.01555	0.0000	-0.00472	0.9666
		β	0.889326	0.0000	1.0068	0.0000	-0.01754	0.9864
	PARCH	ω	6.30E-07	0.3155	0.000166	0.7723	-3.63E-08	0.1127
		α	0.508925	0.0000	0.15	0.0435	0.980411	0.0029
		γ	0.047079	0.1390	0.05	0.7499	0.211674	0.0097
		β	0.48657	0.0000	0.6	0.0000	0.784902	0.0000
		δ	2.560594	0.0000	2	0.0352	1.430173	0.0000

The result is significant at 5% if p<0.05

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

In the estimated ARCH (1), the positive and significant value of the ARCH coefficient infers that the square lagged error terms positively and significantly impacted the present period volatility of maturity returns. While the insignificant ARCH implies no significant influence on the current period volatility of maturity returns.

In the estimated GARCH (1, 1), the significant and positive coefficient of the GARCH term suggested that previous period volatility has a significant effect on the conditional volatility at the present period. The positive ARCH coefficient also revealed that the prior error terms positively and significantly affect the current period volatility and the degree to which volatility reacts to a bond market event is low.

Table 3 Contd
Maximum Likelihood Estimates of GARCH Models with different Conditional Distributions

Maturities	Models	Parameters	Normal		t		GED	
			Estimate	Prob	Estimate	Prob	Estimate	Prob
Ten Years	ARCH	ω	6.66E-05	0.0000	0.007466	0.0000	0.000135	0.0000
		α	1.630139	0.0000	1608.525	0.0000	0.901231	0.0050
	GARCH	ω	1.69E-05	0.0000	5.55E-14	0.3348	4.18E-05	0.4727
		α	0.294314	0.0000	0.59421	0.0000	1.20662	0.4894
		β	0.580585	0.0000	0.529617	0.0000	0.998521	0.0000
	TARCH	ω	1.77E-05	0.0000	2.86E-13	0.4024	4.97E-06	0.7565
		α	0.391766	0.0000	0.410377	0.0000	0.661445	0.4766
		γ	-0.19877	0.0000	0.134979	0.0658	0.97576	0.5708
		β	0.568548	0.0000	0.569543	0.0000	0.99294	0.0000
	EGARCH	ω	-1.18918	0.0000	-0.0213	0.0393	-8.54503	0.1520
		α	0.28451	0.0000	0.269942	0.0010	0.146267	0.2557
		γ	0.053529	0.0001	-0.28164	0.0009	0.014747	0.8989
		β	0.890926	0.0000	1.009022	0.0000	-0.02483	0.9726
	PARCH	ω	9.89E-07	0.5477	-7.91E-10	0.9546	9.84E-05	0.6164
		α	0.303832	0.0000	0.777761	0.0000	0.896879	0.1204
		γ	-0.16719	0.0000	0.18788	0.0000	0.413635	0.0000
		β	0.481876	0.0000	0.656611	0.0000	0.69309	0.0000
		δ	2.617845	0.0000	1.065718	0.0000	1.615019	0.0002
Twenty Years	ARCH	ω	8.59E-05	0.0000	0.012218	0.0000	0.000105	0.0000
		α	0.52146	0.0000	760.4063	0.0003	0.542314	0.0126
	GARCH	ω	1.14E-05	0.0000	4.65E-14	0.5908	1.45E-05	0.6420
		α	0.093785	0.0000	0.579975	0.0000	2.393368	0.5305
		β	0.808301	0.0000	0.594937	0.0000	1.004627	0.0000
	TARCH	ω	1.27E-05	0.0000	1.52E-14	0.5603	2.09E-05	0.1269
		α	0.15875	0.0000	0.742753	0.0015	0.303679	0.1871
		γ	-0.10786	0.0000	0.808336	0.0123	0.029042	0.8621
		β	0.786066	0.0000	0.560966	0.0000	0.601774	0.0000
	EGARCH	ω	-0.95811	0.0000	-0.06989	0.0000	-8.64237	0.0314
		α	0.128003	0.0000	2.754191	0.0000	0.226499	0.2051
		γ	0.070233	0.0000	-2.60266	0.0000	0.094953	0.5308
		β	0.90412	0.0000	1.009749	0.0000	-0.03362	0.9445
	PARCH	ω	7.56E-05	0.1257	1.11E-12	0.9199	3.64E-05	0.7003
		α	0.090263	0.0000	1.222923	0.0255	0.41805	0.0095
		γ	-0.32432	0.0000	0.046668	0.2899	0.199859	0.0991
		β	0.824539	0.0000	0.6127	0.0000	0.57162	0.0000
		δ	1.586863	0.0000	1.6034	0.0000	1.825855	0.0011

The result is significant at 5% if $p<0.05$

From the EGARCH model, the positive and significant ARCH term suggests that the drift of volatility reaction to bond market shocks is significant, and the extent to which it responds to this shock is low. Likewise, prior period volatility affects current period volatility. The insignificant leverage effect term, γ at the 5% level, suggested the nonexistence of leverage effect. A negative leverage parameter indicates an asymmetric reaction for positive returns in the conditional var-iance equation, while a positive leverage parame-ter indicates that bad news leads to increased vol-atility.

In the TGARCH model, the insignificant ARCH term suggested that squared lagged error have no significant effect on the current period volatility and the speed of response of volatility to market shock is high. Likewise, the insignificant GARCH coefficient suggests that prior period variance has no impact on the conditional volatility and it also shows that volatility persistence is high. The positive and insignificant leverage effect suggested that negative shock does not initiate volatility more than an equal level of positive shock.Power ARCH (PARCH) model results shown significant influence in terms of power on the conditional volatility. A significant and positive coefficient from Power ARCH (PARCH) model revealed that the speed of reaction of volatility to market shock is moderate and volatility persistence is high. The significant leverage effect term at a 5% level of significance suggested the presence of leverage.

Table 4. GARCH Models Comparison

Maturities	Models	Distributions	LL	AIC	BIC	HQC
Three Years	ARCH	Normal	2243.238	-5.24032	-5.22365	-5.23394
		t	3013.973	-7.04087	-7.01865	-7.03236
		GED	3659.018	-8.54975	-8.52752	-8.54124
	GARCH	Normal	2605.159	-6.08458	-6.06236	-6.07607
		t	3168.197	-7.39929	-7.37151	-7.38865
		GED	4395.896	-10.2711	-10.2433	-10.2605
	TARCH	Normal	2612.152	-6.0986	-6.07082	-6.08796
		t	3160.375	-7.37866	-7.34531	-7.36589
		GED	3651.24	-8.52688	-8.49354	-8.51411
	EGARCH	Normal	2611.639	-6.0974	-6.06962	-6.08676
		t	3076.826	-7.18322	-7.14988	-7.17045
		GED	3552.816	-8.29665	-8.2633	-8.28388
	PARCH	Normal	2620.077	-6.1148	-6.08146	-6.10203
		t	3213.74	-7.50115	-7.46225	-7.48625
		GED	3851.974	-8.99409	-8.95519	-8.9792
Five Years	ARCH	Normal	2680.957	-6.26423	-6.24756	-6.25784
		t	3176.577	-7.42123	-7.39901	-7.41272
		GED	3922.249	-9.1655	-9.14327	-9.15698
	GARCH	Normal	2524.084	-5.89493	-5.87271	-5.88642
		t	3493.883	-8.16113	-8.13335	-8.15049
		GED	3948.945	-9.2256	-9.19782	-9.21496
	TARCH	Normal	2915.367	-6.80788	-6.78009	-6.79724
		t	3200.88	-7.4734	-7.44006	-7.46064
		GED	5367.165	-12.5407	-12.5074	-12.528
	EGARCH	Normal	2893.005	-6.75557	-6.72778	-6.74493
		t	3443.386	-8.04067	-8.00733	-8.0279
		GED	4261.555	-9.95452	-9.92117	-9.94175

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

	PARCH	Normal	2917.422	-6.81034	-6.777	-6.79758
		t	2424.773	-5.65561	-5.61671	-5.64072
		GED	3953.048	-9.23052	-9.19162	-9.21563
Ten Years	ARCH	Normal	2664.911	-6.22669	-6.21002	-6.22031
		t	3234.702	-7.5572	-7.53497	-7.54869
		GED	4388.087	-10.2552	-10.233	-10.2467
	GARCH	Normal	2851.406	-6.6606	-6.63837	-6.65209
		t	3415.289	-7.97728	-7.9495	-7.96664
		GED	4728.548	-11.0492	-11.0215	-11.0386
	TARCH	Normal	2855.625	-6.66812	-6.64034	-6.65748
		t	3387.437	-7.90979	-7.87645	-7.89703
		GED	4799.638	-11.2132	-11.1799	-11.2004
	EGARCH	Normal	2826.723	-6.60052	-6.57274	-6.58988
		t	3354.696	-7.83321	-7.79987	-7.82044
		GED	4569.406	-10.6746	-10.6413	-10.6619
	PARCH	Normal	2856.896	-6.66876	-6.63542	-6.656
		t	3539.413	-8.26296	-8.22406	-8.24806
		GED	4414.074	-10.3089	-10.2701	-10.2941
Twenty Years	ARCH	Normal	2693.04	-6.29249	-6.27582	-6.28611
		t	3232.339	-7.55167	-7.52944	-7.54316
		GED	4452.416	-10.4057	-10.3834	-10.3971
	GARCH	Normal	2758.415	-6.44308	-6.42085	-6.43456
		t	3340.868	-7.8032	-7.77542	-7.79256
		GED	5453.436	-12.7449	-12.7171	-12.7342
	TARCH	Normal	2762.103	-6.44936	-6.42158	-6.43872
		t	3368.385	-7.86523	-7.83189	-7.85246
		GED	4484.265	-10.4755	-10.4421	-10.4627
	EGARCH	Normal	2751.773	-6.4252	-6.39742	-6.41456
		t	3307.22	-7.72215	-7.68881	-7.70939
		GED	4784.654	-11.1781	-11.1448	-11.1654
	PARCH	Normal	2763.19	-6.44957	-6.41623	-6.4368
		t	3398.876	-7.93421	-7.89532	-7.91932
		GED	4054.719	-9.46835	-9.42945	-9.45345

This study estimated the ARCH, GARCH, TARCH or GJR-GARCH, EGARCH, and PARCH models to determine the best performing model for long term interest rate volatility in Nigeria. To compare all the models in Table 3 and determine the best performing models, we used performance criteria such as LL, AIC, BIC, and HQC.

a. Based on LL, AIC, BIC, and HQC the result shows that GARCH (GED) is the most appropriate model for describing the volatility of a three-year maturity interest rate.
b. Based on LL, AIC, BIC, and HQC the result shows that TARCH (GED) is the most suitable model for modeling the volatility of a five-year maturity interest rate.
c. Based on LL, AIC, BIC, and HQC the result shows that TARCH (GED) is the best model for describing the volatility of a ten-year maturity interest rate.
d. Based on LL, AIC, BIC, and HQC the result shows that GARCH (GED) is the best model for modeling the volatility of the twenty-year maturity interest rate.

It can be concluded that the GARCH (GED) and TARCH (GED) is the best performing model for describing the volatility long term interest rates in Nigeria. GED is the most appropriate innovations distributional assumption for the volatility long term interest rates in Nigeria.

CONCLUSIONS, POLICY IMPLICATION AND RECOMMENDATION

This paper compared some sets of standard GARCH models to evaluate the volatility of long term interest rates in terms of model performance criteria. Besides, all models are estimated assuming Gaussian innovations (Normal) and fat-tailed distributions which included GED and student-t. Based on the model performance criteria, GARCH (GED) is the best performing model for modeling the volatility of three and twenty-year interest rate returns while TARCH (GED) is the best performing model for evaluating the volatility of five and ten-year interest rate returns interest rates in Nigeria. Based on prior study, this study has also shown that asset returns display volatility clustering, leptokurtosis, and asymmetry has the Nigeria bond yields exhibited the characteristics of financial asset volatility.

It is therefore important for the policymakers in Nigeria to note that GARCH (GED) and TARCH (GED) models are more appropriate for modeling long term interest rates in Nigeria. The model will help in the development of tools for effective risk management by monitoring the behavior of bond yields. It will also help the Government in developing policies related to the regulation of long term security. The outcomes of the study are essential in assessing various financial decisions in risk management, asset pricing, portfolio management, assessing leverage and investment decisions.

ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014

Given the level of risk associated with portfolio investment, the study recommends that other works should thoroughly consider variants of GARCH models with innovations distributions for the robustness of results.

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ISSN
2302-1810 (online)
DOI: doi.org/10.21009/wahana.15.014